

Empiric Models of the Earth's Free Core Nutation

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Abstract—Free core nutation (FCN) is the main factor that limits the accuracy of the modeling of the motion of Earth's rotational axis in the celestial coordinate system. Several FCN models have been proposed. A comparative analysis is made of the known models including the model proposed by the author. The use of the FCN model is shown to substantially increase the accuracy of the modeling of Earth's rotation. Furthermore, the FCN component extracted from the observed motion of Earth's rotational axis is an important source for the study of the shape and rotation of the Earth's core. A comparison of different FCN models has shown that the proposed model is better than other models if used to extract the geophysical signal (the amplitude and phase of FCN) from observational data.

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INTRODUCTION

The precession–nutation motion of the Earth's rotational axis in space, or, more precisely, in a celestial or inertial (quasi-inertial) coordinate system, is one of the main components of Earth's rotation. From a practical viewpoint, it is used in the procedure of the transformation of coordinates and vectors of terrestrial and celestial objects between the coordinate system tied to Earth's body and an inertial coordinate system, as required when one solves any astronomical, geophysical, or navigation task involving observations of celestial bodies from Earth's surface and observations of terrestrial objects from space. The motion of the axis of Earth's rotation in space is described by the theory of precession–nutation, which is now accurate to 0.1 mas (IAU2000A model recommended by the International Astronomical Union). However, a comparison of this theory to very long baseline interferometry (VLBI) observations reveals periodic components with amplitudes as high as 0.3 mas. The main component is due to the free nutation of Earth's liquid core (FCN, free core nutation) with a nominal period of about 430 sidereal days. Figure 1 shows the differences between the observed coordinates X and Y of the celestial pole and the position of the pole determined from the IAU2000A theory of nutation according to data from the International VLBI Service for Geodesy and Astrometry (IVS) (Schlueter et al., 2002). Unlike the lunisolar and planetary terms of the IAU2000A theory, this component of Earth's rotation behaves irregularly and cannot be predicted theoretically for any sufficiently long time interval at the required level of accuracy.

Free nutation of Earth's liquid core was predicted as early as more than a century ago as one of Earth's rotational eigenmodes. The frequency of free core nutation is a fundamental quantity, which appears in the equa-

tions of the transfer function that relates the amplitude of nutation oscillations of the absolutely solid Earth and the real Earth. The FCN period and amplitude depend on a number of parameters of Earth's internal structure, such as the constitution and dynamic flattening of the core, its moments of inertia, the differential rotation of the core and mantle, and the dynamic interaction of Earth shells and their rheological characteristics (Dehant and Mathews, 2003; Brzezinski, 1996; 2005; Krasinsky, 2006). However, our knowledge of all these factors is so far insufficient to construct an FCN model of sufficient accuracy. Therefore, observational data and empirical models based on these data provide important information for refining the parameters of theoretical models, putting the further refinement of FCN models at the top of the agenda. For example, the IAU1980 nutation theory developed before the beginning of regular VLBI observations used a hydrostatically equilibrium model of the liquid core, which predicted an FCN period of 460 d, which differs significantly from the modern value of 430 d. On the other hand, the improved accuracy of the empirical models increases the accuracy of the computation of the nutation motion of Earth's rotational axis, which is important for many practical applications. Finally, the need for highly accurate forecasts of the nutation angles, which are necessary for many practical applications, is a factor of no minor importance that stimulates efforts in the development of FCN models.

All FCN models that we consider in this paper are based on the analysis of the differences between the coordinates of the celestial pole determined from VLBI observations and the coordinates determined from the IAU2000A theory. This approach assumes that these differences are mostly due to the contribution of FCN, which is sufficiently justified by the fact that this com-

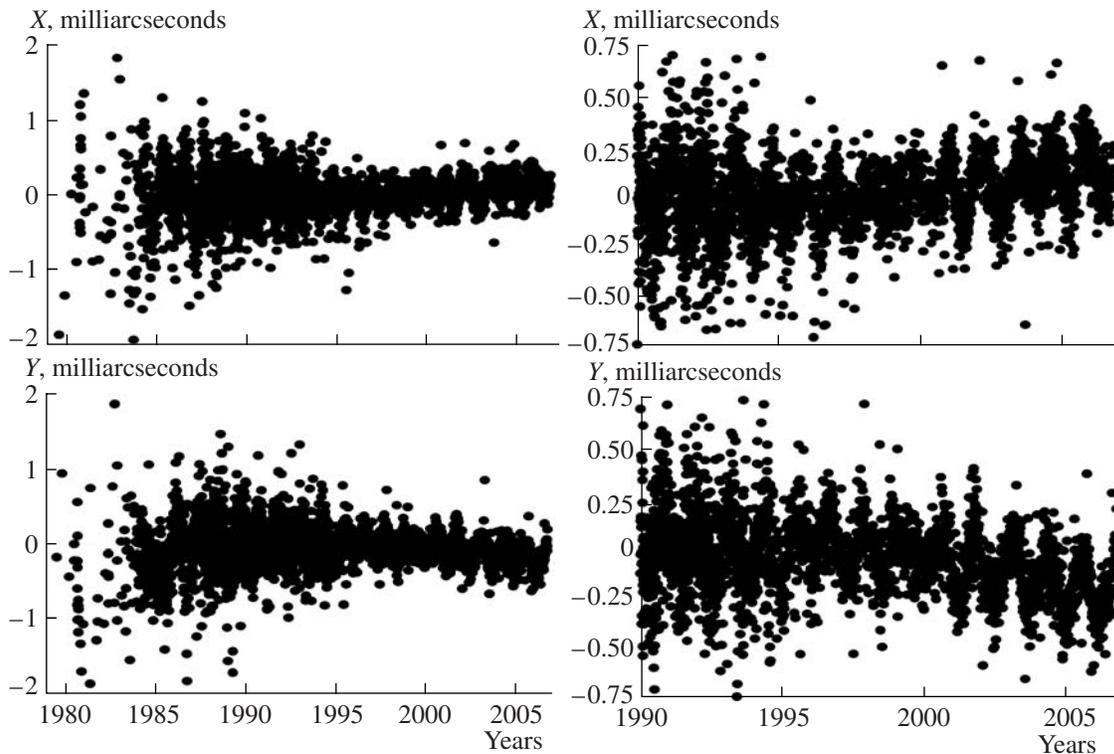


Fig. 1. Differences between the observed coordinates of the celestial pole and its position determined from the IAU2000A theory: the entire observation period (right) and the period from 1990 onward (left).

ponent dominates in the part of Earth's rotational motion that is not modeled by the IAU2000A theory. At the same time, the resonance effect of other nutation terms with periods close to that of FCN may affect the FCN parameters determined from observations (Brzezinski, 1996; Vondrák et al., 2005). This effect must be taken into account in the geophysical interpretation of observational data, but can also be incorporated into the empirical model intended for practical applications.

In this paper, we consider the three most well-known empirical FCN models (Herring et al., 2002; Malkin, 2004; Lambert, 2005, personal communication). We are the first to compare models from the viewpoint of how they represent the observed variations of the FCN amplitude and phase. The results of our comparison have shown that our model is superior to other models in this respect. Our comparative analysis also covers a number of FCN models obtained by simple smoothing of observational data. We found this procedure to ensure minimum residuals in the observations of nutation angles.

EMPIRICAL FCN MODELS

Herring proposed the first FCN model widely used in practice. Successive versions of this model, which differed in that they included new observational data, were incorporated into the KSV and MHB2000 nutation models (Herring et al., 2002). The FCN contribu-

tion to the coordinates of the celestial pole in the model considered can be computed as follows:

$$\begin{aligned} dX &= -A_1 \sin \varphi + A_2 \cos \varphi, \\ dY &= -A_1 \cos \varphi - A_2 \sin \varphi, \end{aligned} \quad (1)$$

where $\varphi = -2\pi r(1 + f_0)t$, and $f_0 = -1.00231810920$ is the FCN frequency, which corresponds to the period of 431.39 sidereal d; $r = 1.002737909$ is the coefficient of transformation from the mean to sidereal time; t is the epoch in Julian days counted from the standard epoch of $t_0 = \text{J2000.0} = 2451545$. The initial amplitudes A_1 and A_2 were computed from an analysis of the differences between the observed and theoretical nutation angles at the epochs of 1979.0, 1984.0–2000.0 with a two-year step, and 2001.41 (the last epoch after which the model is not supported). The analysis was performed on six successive time intervals independently for both components of nutation (Herring et al., 2002). In the case of the practical application of this model, the amplitudes are linearly interpolated to the necessary epoch. We refer to this model as MHB.

Malkin (2004) proposed another FCN model in 2003. We denote it as ZM1. According to this model, the FCN contribution can be computed using the following formulas:

$$dX = A(t) \sin \Phi(t), \quad dY = A(t) \cos \Phi(t), \quad (2)$$

where $A(t)$ and $\Phi(t)$ are the amplitude and phase of FCN, respectively. The variable FCN amplitude can be computed from the differences between the observed and theoretical nutation angles:

$$A(t) = \sqrt{\Delta X^2 + \Delta Y^2}. \quad (3)$$

The initial differences are first smoothed using a Gaussian bandpass filter with a central frequency close to f_0 , which allows cutting off high- and low-frequency components lying outside the FCN frequency band. Smoothing is performed simultaneously with interpolation to equidistant epochs, usually with a ten-day step, to simplify and speed up further computations. Note that Malkin and Terentev (2003) compared the results of computations made with the initial and smoothed differences and found no appreciable differences between them.

We compute the variable phase $\Phi(t)$ as follows. We first perform wavelet-analysis of the differences between the observed and theoretical nutation angles to determine the time variation $\omega(t)$ of the FCN frequency. We then determine the phase of FCN by integrating the frequency:

$$\Phi(t) = \int_{t_0}^t \omega dt + \varphi_0, \quad (4)$$

where φ_0 is the initial phase at the J2000.0 epoch. The phase is computed for the same epochs for which the amplitude has been determined. We finally compute the FCN contribution using formula (2). Note that a characteristic feature of the wavelet analysis is the edge effect, which distorts the data at the extreme subintervals of the time interval considered. Malkin and Terentev (2003) and Shirai et al. (2004) analyzed this effect for the case of FCN. Therefore, although we use all observations made between 1979–2006 to construct our model, we give the final series only for the period 1984.0–2005.0.

In 2004, Lambert proposed an FCN model based on principles similar to those of the MHB model (McCarthy, 2005). We denote this model as SL. The SL model, unlike the MHB model, analyzes the differences between the observed and theoretical nutation angles strictly by two-year intervals, uses both components of nutation simultaneously, and, hence, finds the FCN contribution using the least-squares method in the complex form

$$dX + idY = A \exp(i\omega_0 t) + X_0 + iY_0, \quad (5)$$

where ω_0 is the circular FCN frequency, which corresponds to a period of -430.23 mean d (which is equal to the period of the MHB model), and X_0 and Y_0 are the displacements. As a result, formulas to allow for FCN are derived that are similar to those of the MHB model. Unlike the MHB model, the SL model is continuously updated, including its forecast (Lambert, 2006, per-

sonal communication). The data series begins at 1984.0.

The wavelet analysis of the differences between the observed and model nutation angles plays an important part in the ZM1 model. Two comments can be made in this connection. From the viewpoint of the construction of the model, the choice of the type and parameters of the wavelet and the degree of smoothing of the initial differences are factors that can be varied to achieve the result that best fits the available observations. Here, we consider a variant of the ZM1 model, which yields results that are close to those for the MHB and SL models as shown below. The second comment concerns the geophysical interpretation of the model. From the mathematical viewpoint, the phase variation obtained from the analysis can be a result of the variation of the period of nutation oscillations. Strictly speaking, we have the variations of both the period and phase that can be separated only by invoking a geophysical analysis. A number of authors, e.g., Hinderer et al. (2000) and Zharov (2005), show that the FCN period remains constant within ± 2 d. Therefore, the dependence found is most likely due to phase variations, which can be correlated with other geophysical observations. Shirai et al. (2005) give an example of such a correlation. Thus, the variations of the amplitude and phase determined based on empirical models provide material of greatest interest for further geophysical interpretation.

COMPARISON OF THE MODELS

We compared the FCN models described above based on the following two criteria. In the first test, we compared the models using the reduction of residuals in the observations of the nutation angles. In the second test, we compared the quality of the representation of the temporal variations of the FCN amplitude and phase.

For completeness, we also included into our list of models to be compared the FCN series obtained by simple smoothing of the observed differences between the observed and model nutation angles. This is actually the series (we denote it as ZM2) obtained at the first stage of the construction of the ZM1 model. Although it is not a model in the true sense of the word, it is nevertheless a good supplement to the IAU2000A nutation model and allows the accuracy of the modeling of nutation to be substantially improved in the case of the transformation between terrestrial and celestial coordinate systems. It is also easy to forecast both forward and backward and can, therefore, be applied for operational and forecasting tasks and for processing old observations. This series is currently computed for the period from 1976 to 2008 and is permanently updated.

We test the reduction of observational residuals in the case of the use of FCN models by analyzing their spectrum and computing their root-mean-square value. Figure 2 shows the spectrum of residuals before and

after the application of the ZM1 model. As is evident from the figure, the spectral peak corresponding to the frequency of this nutation oscillation virtually disappears after the application of the FCN model. Other FCN models described in this paper yield similar results.

The table gives the numerical data on the reduction of residuals after the application of different FCN models. This table gives the weighted root-mean-square differences between the nutation angles determined from VLBI observations (the combined IVS series) and corrected by the FCN models and the angles given by the IAU2000A model. The first line of the table gives the data for all observation sessions from January 1984 through June 2001, i.e., for the maximum interval of dates covered by all four compared models. The second row gives the data for 2003–2004, i.e., for the last two years when models ZM1, SL, and ZM2 were determined. We computed the root-mean-square differences in two variants: “as is” and after the elimination of the constant shift. The difference between these two variants is almost negligible over the entire observation interval, but it is sufficiently large for the last two years. This is explained by the fact that the differences between the observed and modelled nutation angles show an appreciable trend in recent years as is evident from Fig. 1. Maybe this trend will disappear after the use of the new P03 precession model, which was recommended by the last General Assembly of the International Astronomical Union in August 2006.

It is evident from the data listed in the table that the trend affects the residuals for the MHB, ZM1, and SL models, where the trend is eliminated at the stage of the construction of the models. At the same time, the trend has virtually no effect on the residuals for the ZM2 model, because this FCN series was computed without removing the trend. These results lead us to an important conclusion about the domains for the optimal application of different models. The first three models are best suited for the geophysical interpretation of the observed data on the free nutation of the liquid core, whereas the ZM2 series yields the most accurate result in practical computations involving transformations of the coordinate system.

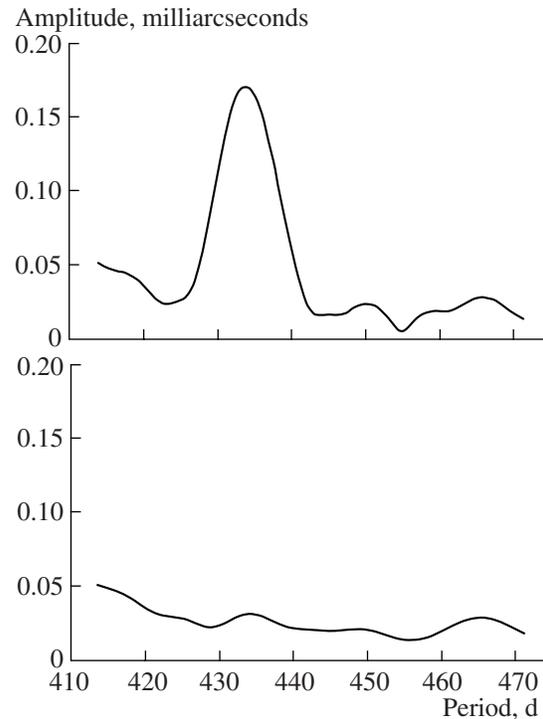


Fig. 2. The spectrum of residuals before (top) and after (bottom) correction for the FCN model.

We now compare the MHB, ZM1, and SL models from the viewpoint of the most interesting geophysical data—variations of the FCN amplitude and phase. For any FCN model given in the form of a series of corrections to the *X* and *Y* coordinates of the celestial pole, the FCN amplitude can be computed using formula (3), and the phase can be determined as $\text{atan}(\arctg(X/Y))$. Figure 3 shows the results. The data presented show that all models exhibit similar variations in the amplitude and phase of FCN, but the variations yielded by the ZM1 model are smoother and, thus, apparently better reproduce the real variations in Earth’s rotation. It is evident from the principles of the construction of the FCN models that the ZM1 model yields almost continuous in-time determination of FCN parameters with any preset step in contrast to the previous models,

Root-mean-square residuals in milliseconds of arc

Interval of dates	Raw differences					After removing the trend				
	FCN model					FCN model				
	No model	MHB	ZM1	SL	ZM2	No model	MHB	ZM1	SL	ZM2
1984–2001	184	155	156	155	143	184	155	156	155	143
2003–2004	156	–	143	141	89	130	–	100	102	88

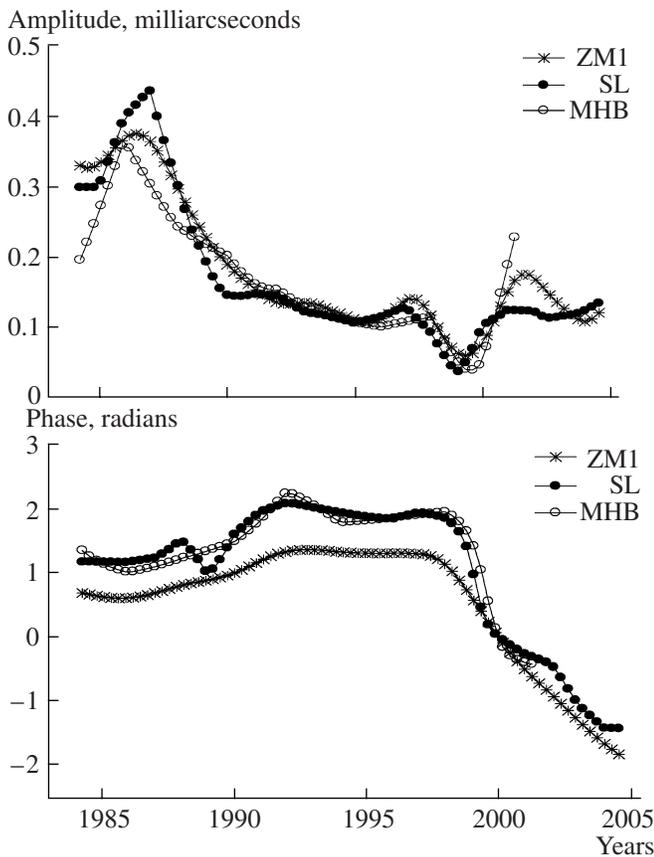


Fig. 3. Variation of the FCN amplitude and phase (with the linear trend removed) for different models.

which determine these parameters with a two-year time step, resulting in glitches in FCN parameters, which are immediately apparent in Fig. 3 for the MHB and SL models.

CONCLUSIONS

In this paper, we compared several models of the free core nutation of Earth (FCN). We have shown that the use of an FCN model substantially improves the accuracy of the modeling of nutation. A comparison of different FCN models has shown that our ZM1 model gives better results if used to extract the geophysical signal (the amplitude and phase of FCN) from observational data, whereas the ZM2 series best allows for the FCN contribution when solving practical tasks involving the transformation between coordinate systems.

In this paper, we analyzed empirical FCN models without considering their physical meaning. Many authors analyzed the physical FCN models and its excitation based on the available knowledge on Earth's structure and on the interaction of the outer and inner Earth layers (Mathews and Shapiro, 1996; Brzeziński and Petrov, 1999; Shirai and Fukushima, 2001a; 2001b; Herring et al., 2002; Dehant and Mathews, 2003;

Zharov, 2005; Krasinsky, 2006); however, the analysis of these models lies beyond the scope of this work.

Note also that, here, we only analyzed the principal mode of Earth's rotational motion, which corresponds to the free nutation of the Earth's core with a nominal period of about 430 d. At the same time, a number of authors have confidently identified a second oscillation with a close frequency of about 410–420 d (Malkin and Terentev, 2003; Schmidt et al., 2005), which can also be physically explained in terms of a more complex two-layer model of the liquid core (Krasinsky, 2006; Krasinsky and Vasilyev, 2006). If the oscillations with a close frequency are real, they must be incorporated into all empirical models based on the approximation of the observed differences of nutation angles by the official model. At the same time, an analysis of the two-component FCN model is one of the most interesting directions for further studies, which will make it possible to construct a model that would better approximate the physical properties of Earth and improve the accuracy of the forecast.

Note, in conclusion, that the ZM1 and ZM2 models are available via the Internet site of the Central Astronomical Observatory at Pulkovo (Pulkovo Observatory) (http://www.gao.spb.ru/english/as/ac_vlbi/).

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