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ORIGINAL ARTICLE

On the accuracy assessment of celestial reference frame realizations

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Abstract The scatter of the celestial pole offset (CPO) time-series obtained from very long baseline interferometry (VLBI) observations is used as a measure of the accuracy of celestial reference frame (CRF) realizations. Several scatter indices (SI), including some proposed for the first time, are investigated. The first SI is based on residual analysis of CPO series with respect to a free core nutation (FCN) model. The second group of SIs includes Allan deviation and its extensions, which allow the treatment of unequally weighted and multidimensional observations. Application of these criteria to several radio source catalogues (RSCs) showed their ability to perform a preliminary assessment of the quality of each RSC. The 2D Allan deviation estimate seems to be the most sensitive measure. The proposed extensions of Allan deviation, weighted and multidimensional, can also be used for the statistical analysis of other time-series.

Keywords Reference systems · Celestial reference frame · VLBI · Statistical data analysis · Accuracy assessment

1 Introduction

Very long baseline interferometry (VLBI) is the base technique for the realization of the International Celestial Reference Frame (ICRF) given as a set of radio source coordinates (Ma et al. 1998). Improvement of the ICRF's accuracy is one of the primary tasks of the International VLBI Service for Geodesy and Astrometry (IVS), and the assessment of the ICRF's accuracy is of primary importance in its improvement (Schlüter and Behrend 2007).

Many studies of the accuracy of the ICRF have been performed. Some authors have investigated the time behavior of radio source positions (e.g., Gontier et al. 2001; Feissel-Vernier 2003; MacMillan and Ma 2007). Others have studied the accuracy of different CRF realizations by means of investigation of stability of the coordinate axis (e.g., Arias et al. 1988; Arias and Bouquillon 2004; Feissel-Vernier et al. 2006). However, existing methods allow us to investigate only *differences* between celestial reference frame (CRF) realizations or, in other words, radio source position catalogues (RSCs). There is no evident and generally accepted methods to assess the *absolute* RSC, including ICRF, accuracy.

For nearly the past three decades, VLBI uniquely provides highly accurate measurements of the movement of the Earth's rotation axis in space, precession and nutation. Currently, the most accurate precession-nutation model recommended by the International Astronomical Union (IAU) comprises the IAU2000A nutation model and the IAU2006 precession model (see McCarthy and Petit 2004, Chap. 5 and cited papers for details), hereafter referred to as IAU2000A.

Observed corrections to the precession-nutation model are called celestial pole offsets (CPOs). One of the main factors, which affect the quality of the CPO obtained from VLBI observations, is the accuracy of the RSC used during data processing. In this paper, possible criteria to assess the RSC accuracy through its impact on the CPO time-series are considered.

The criteria tested in this study are based on investigation of the scatter of CPO time-series obtained from VLBI observations as a measure of the accuracy of the CRF realizations. Several scatter indices (SIs), including some proposed for the first time, were applied to the RSC obtained in the framework

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of the joint project of the International Earth Rotation and Reference Systems Service (IERS) and the IVS on the next ICRF realization (Ma 2005), as well as the RSC recently submitted by IVS Analysis Centers (ACs) to the IVS Data Centers. The latest ICRF realization, ICRF-Ext.2 (Fey et al. 2004), was also tested. Based on the obtained results, one can make a conclusion on the ability of various SIs to estimate the accuracy (quality) of the CRF realizations provided by VLBI ACs.

2 Scatter indices

Investigation of the scatter of a geodetic time-series is a powerful tool for the assessment of its quality and statistical characteristics (e.g., Gontier et al. 2001; Gambis 2004; Trenkamp et al. 2004; Feissel-Vernier and Le Bail 2005; MacMillan and Ma 2007). The scatter of the time-series can be computed in different ways. The term "scatter index" (SI) will be used to distinguish different methods.

2.1 Residuals of CPO series

Both precision and accuracy of the CPO estimates directly depend on the accuracy of radio source positions used during VLBI data processing (Sovers et al. 1998). There is no external *absolutely accurate* reference series that may be used for comparison, since only the VLBI technique provides highly accurate CPO measurements.

For many years, the scatter of Earth orientation parameters (EOPs) time-series is widely used to compare EOP series computed by various ACs. Here, this will be applied to the comparison of CRF realizations. A similar test was used in Feissel-Vernier et al. (2006), where they compared several RSCs obtained in the course of their work using the weighted root-mean-square (WRMS) of the differences with respect to the IAU2000A model.

However, it seems that such a test can be improved using IAU2000A supplemented with a FCN model as a reference. In this case, the residuals between VLBI nutation series and the model (CPO) become smaller by a factor of 2–3, and the test should be more sensitive. Thus, the IAU2000A model was used with the addition of the observed corrections computed by means of smoothing the differences between the IVS combined EOP series and the IAU2000A model.[1] Since the FCN is the largest part of the observed corrections, for conciseness, hereafter, the latter will be referred to as the FCN contribution.

However, this SI cannot provide an independent estimate of the CPO scatter, since any FCN model obtained from a comparison of observations and the IAU2000A model depends on results of VLBI data processing, and thus, such an estimate implicitly depends on underlying CRF realizations.

2.2 Allan deviation

The Allan variance (AVAR) was originally developed for investigation of noise parameters of frequency standards (Allan 1966; Allan et al. 1991). In the last years, AVAR-based methods are often applied to the analysis of geodetic and geodynamic time-series. For example, the IERS EOP Product Center at the Paris Observatory employs this method for the assessment of statistical characteristics of the EOP series (Gambis 2002, 2004). Furthermore, this method is widely used to investigate stochastic properties of station coordinates and baseline time-series (e.g., Malkin and Voinov 2001; Roberts et al. 2002; Le Bail and Feissel-Vernier 2003; Le Bail 2006). Gontier et al. (2001) and Feissel-Vernier (2003) have used the AVAR to analyze the noise in the time-series of radio source positions. An advantage of using the AVAR to investigate the noise of time-series is its independence, in most of the cases, from the systematic errors, trends and low-frequency variations.

Usually, the square root of the AVAR, the Allan deviation (ADEV), is used in geodetic time-series analysis. The ADEV estimate for time series of measurements (observations) $y_1, y_2, \ldots, y_n$ is given by (Allan 1966)

$$\mathrm{ADEV} = \sqrt{\frac{1}{2(n-1)} \sum_{i=1}^{n-1} (y_i - y_{i+1})^2}\,, \qquad (1)$$

where n is the number of measurements.

Generally, Eq. (1) is valid for equally weighted measurements. However, in geodetic and astrometric practice, we mostly deal with measurements of different precision. In this case, we have the measurements $y_1, y_2, \ldots, y_n$ with the associated uncertainties $s_1, s_2, \ldots, s_n$. To account for the difference in precision of the measured values, they should be weighted. In our case, to treat unequally weighted data, an extension to the ADEV, weighted Allan deviation (WADEV), is proposed and given by

$$\begin{aligned} \mathrm{WADEV} &= \sqrt{\frac{1}{2p} \sum_{i=1}^{n-1} p_i (y_i - y_{i+1})^2}\,, \\ p &= \sum_{i=1}^{n-1} p_i\,, \\ p_i &= (s_i^2 + s_{i+1}^2)^{-1}\,, \end{aligned} \qquad (2)$$

where p_i are the weights. Figure 1 displays an example of actual weekly series of a station coordinates, which illustrates that the difference between ADEV and WADEV estimates may be significant.

[1] http://www.gao.spb.ru/english/as/persac/fcn2.dat.

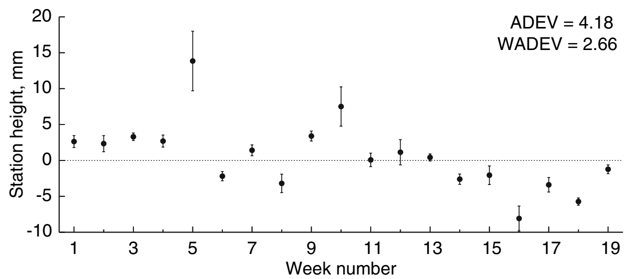

Fig. 1 An example of estimation of classical (ADEV) and weighted (WADEV) Allan deviation for unequally weighted measurements: a case of weekly series of a station height

When the statistical parameters of astrogeodetic time-series are investigated, we often deal with multidimensional values, e.g., terrestrial coordinates and/or velocities (3D or 6D), celestial coordinates and/or proper motions (2D or 4D). Further extension of the WADEV estimator, weighted multidimensional Allan deviation (WMADEV) is useful for analysis of such types of time-series. In this case, we can consider multidimensional measured values $y_i = (y_i^1, y_i^2, \ldots, y_i^k)$ with the associated k-dimensional uncertainties $s_i = (s_i^1, s_i^2, \ldots, s_i^k)$.

The main idea of the WMADEV extension of the classical Allan deviation is the consideration ADEV as a statistics of the differences between the adjacent measurements, which can be taken as the Euclidean length between the adjacent measured points considered as the coordinates in some space, one-dimensional in the classical case, and k-dimensional in the extended one. Thus, we can define the k-dimensional Allan deviation as

$$\begin{aligned} \mathrm{WMADEV} &= \sqrt{\frac{1}{2p}\sum_{i=1}^{n-1} p_i\, d_i^2}\,, \\ d_i &= |y_i - y_{i+1}|\,, \\ p &= \sum_{i=1}^{n-1} p_i\,, \\ p_i &= \left(\sum_{j=1}^{k}\left\{\left[(y_i^j - y_{i+1}^j)/d_i\right]^2 \left[(s_i^j)^2 + (s_{i+1}^j)^2\right]\right\}\right)^{-1}, \end{aligned} \tag{3}$$

where $|\ldots|$ denotes the Euclidean norm.

The expression for the weights p_i is derived from the error propagation law. One can see, it has a singular point in the case of $d_i = 0$, i.e. of equal adjacent measurements. Hence, this case require special treatment, e.g. assigning unit weight for (near-)zero d_i. To avoid the singularity, a simplified formula for computing p_i is

$$p_i = \left(\sum_{j=1}^{k}\left[(s_i^j)^2 + (s_{i+1}^j)^2\right]\right)^{-1}. \tag{4}$$

Table 1 IVS analysis centers providing the RSCs used in this study

Abbreviation	Full name
AUS	Geoscience Australia
BKG	Bundesamt für Kartographie und Geodäsie, Germany
DGF	Deutsches Geodätisches Forschungsinstitut, Germany
GSF	NASA Goddard Space Flight Center, USA
JPL	Caltech/NASA Jet Propulsion Laboratory, USA
MAO	Main Astronomical Observatory of the National Academy of Sciences of Ukraine
SHA	Shanghai Astronomical Observatory, China
USN	US Naval Observatory, USA

No significant difference between results of WMADEV estimates computed with weights given by Eqs. (3) and (4) was found during processing of real data.

3 Comparison of catalogues

For this study, several CPO time-series were computed with the OCCAM software in the Kalman filter mode. For all computed CPO series, the processing options were kept the same, except different RSCs were used. During the processing, station and source positions were kept fixed, and only EOPs (universal time and terrestrial and celestial pole coordinates), clocks and troposphere parameters, including troposphere gradients (one value per session), were estimated. 504 IVS R1 and R4 VLBI sessions observed during the period 2002.0–2007.0 were processed. Then, the SI described above was computed for all the series.

Three groups of VLBI CRF realizations were included. First, eight RSCs obtained in the framework of the joint IERS-IVS project mentioned above.[2] The RSCs were provided by eight IVS ACs listed in Table 1.

The organizers of the IERS-IVS project requested the participating ACs to submit two versions of RSCs obtained using all available data since 1979 (version 1), and the data from 1900 only (version 2). The intention was to investigate the impact of data selection on CRF realization.

Six RSCs submitted by IVS ACs in 2006–2007 in the framework of their routine operations[3] were then used. The latest ICRF realization, ICRF-Ext.2 (Fey et al. 2004), was also included in this comparison. Results of this test are presented in Table 2.

From Table 2, the accuracy of all the compared RSCs as estimated by this method is generally about the same level. Nevertheless, some discrepancies can be seen. In particular, the ICRF-Ext.2 shows the worst result, i.e largest

[2] ftp://cddis.gsfc.nasa.gov/vlbi/ivs-special/icrf-next/.

[3] ftp://cddis.gsfc.nasa.gov/vlbi/ivsproducts/.

Table 2 Scatter indices for the CPO time-series obtained with different RSCs

Catalogue	FCN	ADEV	WADEV	WMADEV
IERS/IVS project catalogues, 1979–2005				
AUS1	99	113	104	150
BKG1	96	111	104	147
DGF1	96	113	104	148
GSF1	96	111	103	146
JPL1	98	113	105	149
MAO1	96	113	104	147
SHA1	96	114	104	148
USN1	96	114	104	147
IERS/IVS project catalogues, 1990–2005				
AUS2	97	113	104	148
BKG2	95	112	103	146
DGF2	96	111	103	146
GSF2	95	111	103	146
JPL2	96	110	102	145
MAO2	95	111	103	145
SHA2	96	112	104	147
USN2	95	111	102	145
Latest catalogues				
AUS2006B	97	112	103	147
BKG2006C	96	112	103	146
CGS2006A	95	110	102	144
GSF2007A	96	111	103	146
IAA2006A	97	114	106	150
USN2006C	95	110	102	144
ICRF-Ext.2	99	117	108	154

FCN column shows the WRMS of the differences between CPO series computed with given RSC and IAU2000A+FCN reference series, ADEV and WADEV columns contain classical and weighted Allan deviation estimates. The values presented in the FCN, ADEV and WADEV columns are computed as the mean of the corresponding independent estimates for the x_p and y_p celestial pole coordinates. The WMADEV column contains 2D estimates computed with the Eq. (3), $k = 2$, for both x_p and y_p coordinates. Unit: μas

scatter, most probably because in ICRF-Ext.2, the 212 *defining* source positions were kept the same as in the first ICRF realization. In the analyzed data, 186 *defining* sources were observed of total number of 507 (36.6%), and there were 307155 observations of *defining* sources of total number of 953637 (32.2%). Thus, positions of the *defining* sources used for EOP computations have a large impact on the result.

Since using ICRF yields the largest scatter of the CPO series as compared to the other catalogues tested, one can conclude that it might be inadvisable to keep coordinates of the *defining* ICRF sources in successive ICRF realizations. In particular, such a conclusion follows from comparison of the SI values obtained for the ICRF-Ext.2 and USN RSC. Both the RSCs were computed at the USNO using the same software and the similar processing options, and the main difference in the data processing strategy was handling of the ICRF *defining* sources, whose positions were adjusted under the No-Net-Rotation condition in the USN catalog and were kept to the first ICRF in the ICRF-Ext.2 (Alan Fey, 2006, private communication).

From our test, one can see that the catalogues of version 2 constructed using only the observations made from 1990.0 on show small but steady improvement of the accuracy. This effect can also be explained by a larger weight of the IVS R1/R4 sessions in the RSC of version 2 with respect to the RSC of version 1. On the other hand, previous studies (Malkin 2004a,b) have shown that both EOP and precession parameters derived from VLBI observations show smaller uncertainty when only data from 1990 on are used for analysis.

Comparing the SIs considered in this test, the 2D WMADEV estimate seems to be the most sensitive criterion, which can be seen from the ratio of the smallest and largest SI values in each column of Table 2, i.e. for different RSCs. In other words, the WMADEV shows the most scatter among the different CRF realizations. The same ratio comparison shows that 'FCN' SI is the least sensitive.

4 Conclusion

In this paper, the scatter of CPO series obtained from VLBI observations was used as a measure of the accuracy of the CRF realizations. Several scatter indices (SIs) have been investigated.

The first SI is based on analysis of residuals of CPO series with respect to the IAU2000A precession-nutation model supplemented with the FCN contribution. It turned out to be less sensitive than the other SIs tested. Besides this, the SI cannot be considered as fully independent, since both the IAU2000A model and FCN contribution determined by previous VLBI data analysis rely on CRF realizations. Conversely, the SIs based on Allan deviation and its extensions, weighted Allan deviation (WADEV) and weighted multidimensional Allan deviation (WMADEV), proposed in this paper, provide an independent estimate of the quality of CRF realizations.

The application of these criteria to several RSCs has shown their ability to perform a preliminary assessment of the RSC quality. The WMADEV seems to be most sensitive to differences in different CRF realizations (see Table 2).

Proposed extensions of the classical Allan deviation estimator, WADEV and WMADEV, can also be used for statistical analysis of other geodetic time-series, such as EOP, station positions and troposphere parameter time-series. Both WADEV and WMADEV estimates can be also applied, like the classical ADEV estimate, for different sampling intervals as well as overlapping intervals for more detailed investigation of the various time-series noise (see, e.g., Gambis 2002, 2004; Feissel-Vernier 2003; Williams 2003; Le Bail 2006).

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