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In August 2009 the Siberian State Research Institute of Metrology (SNIIM) celebrated its 65th anniversary. Articles reflecting the main areas of activity at the Institute by leading members of its staff have been published in the two preceding issues of this journal.
In this issue we conclude this set of articles.

HIGH PRECISION PREDICTION OF UNIVERSAL TIME BASED ON 100-YEAR DATA

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A new method is proposed for predicting universal time based on constructing a general harmonic model for the earth's rotation and using a modified autoregression technique. A comparison of the predictions made at the Siberian State Research Institute of Metrology (SNIIM) and the International Earth Rotation and Reference Systems Service (IERS) during 2008–2009 shows that the former are more accurate. The efficiency of the new prediction technique for time support of the GLONASS system is demonstrated.
Key words: *earth's rotation, prediction, universal time, harmonic model, time series, autoregression.*

Studies of the rotation of our planet have always been of fundamental importance for many scientific problems, especially in the applied disciplines with which most geosciences are associated. Thus, in geophysics this helps in avoiding false hypotheses during the construction of models for the earth's internal structure, in meteorology – in refining global climate change models, and in geodynamics – in determining the parameters of the earth's orientation relative to the inertial space of the stars.

The rotation of the earth, i.e., its instantaneous angular orientation in the inertial space, is represented by a set of elements known as the parameters of the earth's rotation. These parameters typically include the "instantaneous phase of the earth's rotation," given in the form of the difference in scales between universal (*UT1*) and coordinated universal (*UTC*) time: $dUT1 = UT1 - UTC$. This parameter is also referred to as the clock correction or simply as the correction to universal time *UT1*. The other parameters of the earth's rotation include the polar coordinates X_p, Y_p, which characterize the position of the axis of rotation in the body of the earth, and the corrections to the precession and nutation angles, X_c, Y_c, which represent the deviations in the instantaneous direction of earth's axis of rotation in space from that calculated by the accepted theory.

Highly accurate prediction of the parameters of the earth's rotation acquired a special significance in the early 1980s when Russia and the USA launched global satellite navigation systems (GLONASS and GPS). The development of these systems requires constant improvement in the on-board apparatus and programming support. In particular, the placement in orbit

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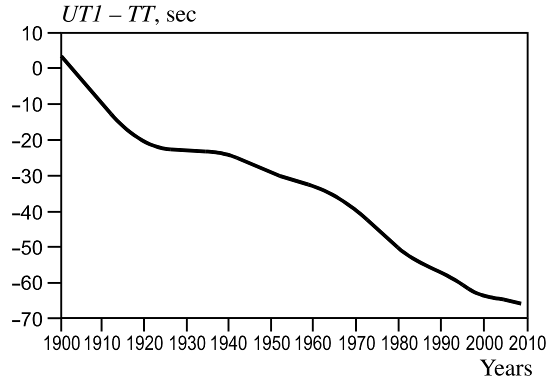

Fig. 1. The variation in the differences *UT1–TT* during the period from 1900 to 2009.

of new spacecraft in the GLONASS-M and GLONASS-K series has led to a requirement for better quality ephemeris and time service by GLONASS, which would not be possible without highly accurate prediction of the parameters of the earth's rotation that serve as initial conditions for the differential equations of motion of the spacecraft. In other words, in order to calculate the exact orbits of these spacecraft it is necessary to have exact predictions of the parameters of the earth's rotation. This is especially true of the periods, when autonomous operation of the navigation systems is required.

Universal time is traditionally considered to be the parameter which is most difficult to predict. This is because of the complex way it has varied over the 350-year observational period. Figure 1 is a plot of the variation in the difference between the nonuniform scale of universal time *UT1* and the uniform scale of earthbound dynamic time *TT* (ephemeris *TE* from 1900 to 1956 and atomic *TA1* from 1956–2009); this illustrates the complicated, nonuniform rotation of the earth during this period.

Figure 1 shows that over the last 100 years there have been at least 6 multi-year (10 or more years) trends in universal time with different magnitudes and directions. Piecewise-continuous variations of this type, as well as similar shorter variations over 1–2 years, are currently regarded as essentially unpredictable.

Methods of Predicting Universal Time. In practice, the most accurate and stable results in predicting the motion of mechanical systems are obtained using physical-mathematical models calculated under conditions of a balance condition for the forces and momenta acting on the system being studied. However, it is extremely difficult to apply this type of model to the rotating earth because of the complicated, unknown distribution of mass within the earth's interior and because of the variety of forces acting on it. Thus, use is traditionally made of empirical models in the form of sets of parametric families of known functions or operators which serve for interpolating the results of measurements. In this case, there are some natural limitations on improvements in the accuracy of predicting the parameters of the earth's rotation; these are associated both with the nonstationarity in the series of the known data and with the presence of random and not entirely random noise in these data. The main sources of this noise are imperfections in the measurement apparatus and the impossibility of separating the process being analyzed from the interaction with a multitude of other processes which form an almost random background. Thus, in order to obtain converging solutions at the ends of the interpolation intervals, one must perform multiple computer simulations using different combined techniques of mathematical statistics.

The existing methods for predicting *dUT1* [1, 2] are based on considering the secular-trend and periodic components of the variations in the earth's rotation velocity. The periodic variations are caused by seasonal variations in the climate and ocean tides. The seasonal variations in universal time are usually approximated by the sum of the annual and semiannual waves, whose parameters, like those of the secular trend, are found empirically by means of Fourier analysis or a least squares technique from the known values of *dUT1* over the previous 2–5 years [1, 4], while the parameters of the tidal variations are calculated using a physical-mathematical theory of the motion of the moon and sun. The final prediction, which takes partial account of the stochastic component of the series for the parameters of the earth's rotation, is usually formulated using techniques of autoregression, with integration over a sliding mean or simple exponential smoothing. In recent years, the methods of mean quadratic collocation, singular spectral analysis, and artificial neuron networks [1, 2] have also come into use. Other predictive techniques, including adaptive ones [3], are also used.

The major deficiencies of these methods are the dependence of the results on the range of the data and on the criteria of adequacy employed for estimating the parameters of the predictive model. Thus, in the process of predicting, an informal, essentially empirical approach for choosing one or more of the model parameters is often required of the investigator. Using data sets from only the last 4–6 years will not provide adequate accounting for the irregular, prolonged variations in universal time shown in Fig. 1.

At present, many Russian and foreign services and scientific institutions are concerned with the development and improvement of methods for predicting the parameters of the earth's rotation. The official provider of predictions of the parameters of the earth's rotation in Russia is the State Service for Time, Frequency, and Determining the Earth's Rotation Parameters (GSVCh), under the Russian Technical Regulation Agency (Rostekhregulirovanie). The International Earth Rotation (and Reference Systems) Service (IERS) provides its predictions daily on the internet at the site of the US Naval Observatory (USNO).

It should be noted that the accuracy of short-term (over 10–15 days) predictions of universal time at the IERS and GSVCh has increased markedly in recent years. Thus, we estimate that in 2008 the mean square errors over an interval of 15 days were about 1.5 and 2.0 msec, respectively. These results, however, are not fully satisfactory for the further improvement of the GLONASS ephemeris-time service, which requires a yearly average mean square error of no more than 1 msec.

Brief Description of the Proposed Method. A method has been developed at the Siberian State Research Institute of Metrology (SNIIM) for predicting universal time in which the secular-trend variations are represented by a multicomponent harmonic model over an interval of at least 100 years.

The predictions are calculated in the following stages in this model:

In the *first stage*, corrections owing to the tidal components of the earths rotation owing to the periodic gravitational effect of the moon and sun are eliminated from the time series of measurements of the parameters of the earth's rotation. These corrections for *UT1* and the polar coordinates X_p, Y_p are calculated by the method adopted by the IERS [5].

In the *second stage*, the problem of parametrically identifying mathematical models for time series of the parameters of the earth's rotation is solved. The models are linear combinations of harmonic functions with different amplitudes, periods, and phases.

Long-period components of the nonuniformities in the earth's rotation with periods greater than 20 years in the intervals of a sample to 100 years initially show up in time series of the parameters of the earth's rotation to be identified. Here the periods are chosen by sorting with a specified grid size, and the amplitudes and phases are found by a least squares method using the identity

$$A_j \sin(\omega_j t_i + \varphi_j) = B_j \sin \omega_j t_i + C_j \cos \omega_j t_i.$$

Then the parameters of the remaining, shorter-period components are determined. Here the search for and improvement in their periods are carried out using an original algorithm for summing the half periods of the unknown harmonic components. Summing the values of the time series being analyzed is done with opposite signs for the even and odd segments corresponding to the unknown half periods of the harmonics. Maximum values of the adequacy criterion for the short period component are obtained by varying the lengths of the half periods. After this, a least squares method is used with the found period to determine the amplitude and phase of the harmonic component, which is eliminated from the time series to be analyzed, and the search for neighboring harmonic components is continued on the resulting residual. The search for short period components is continued until a minimum level of the residue of the time series with the excluded short period components is obtained.

In the *third stage*, more precise predictions of the parameters of the earth's rotation are obtained by the method introduced in [6]. This method is based on a probabilistic-statistical accounting for the short time future variations in universal time based on the character of the random perturbations over the interval of known data for the near prehistory.

In the *final stage*, the predictions obtained in the previous stage are improved using a modified autoregression procedure where, instead of the values x of any of the parameters of the earth's rotation from the predicted series, one uses its differences to the sixth order, inclusively. Then the predicted values can be expressed in terms of the sum of a reference value x_{ref} and decreasing corrections $x_i^{(n)}$, where n is the order of the difference, while in the standard autoregression procedure a series of close values of x_i are used in a series with alternating signs to compute the prediction. In our case, the prediction at the $(i + 1)$st step are calculated using the formula

$$x_{i+1} = a_0 x_i + a_1 x_{i-1} + a_2 x_{i-2} + \ldots + a_{k-1} x_{i-k} + \ldots;$$

$$b_0 \Delta'_i + b_1 \Delta'_{i-1} + b_2 \Delta'_{i-2} + \ldots + b_{k-2} \Delta'_{i-k+1} + \ldots;$$

$$c_0 \Delta''_i + c_1 \Delta''_{i-1} + c_2 \Delta''_{i-2} + \ldots + c_{k-3} \Delta''_{i-k+2} + \ldots;$$

$$d_0 \Delta'''_i + d_1 \Delta'''_{i-1} + d_2 \Delta'''_{i-2} + \ldots + d_{k-4} \Delta'''_{i-k+3} + \ldots,$$

where a_j, b_j, c_j, and d_j are the autoregression coefficients between the predicted and reference dates and the values of their derivatives ($j = 1, \ldots, k$); k is the dimensionality of the autoregression matrix; $\Delta'_i = x_i - x_{i-1}$ are the first differences; $\Delta''_i = \Delta'_i - \Delta'_{i-1}$ are the second differences; and, $\Delta'''_i = \Delta''_i - \Delta''_{i-1}$ are the third differences.

The autoregression coefficients are found by solving the following system of equations:

$$\begin{cases} R_{11} a_0 + R_{12} b_0 + R_{13} c_0 + R_{14} d_0 + \ldots + R_{1k} d_{k-3} = R_{01}; \\ R_{21} a_0 + R_{22} b_0 + R_{23} c_0 + R_{24} d_0 + \ldots + R_{2k} d_{k-3} = R_{02}; \\ \ldots \\ R_{(k-1)1} a_0 + R_{(k-1)2} b_0 + R_{(k-1)3} c_0 + R_{(k-1)4} d_0 + \ldots + R_{(k-1)k} d_{k-3} = R_{0(k-1)}; \\ R_{k1} a_0 + R_{k2} b_0 + R_{k3} c_0 + R_{k4} d_0 + \ldots + R_{kk} d_{k-3} = R_{0k}, \end{cases}$$

where $R_{ij} = \langle (x_i - \langle x_i \rangle)(x_j - \langle x_j \rangle) \rangle$ are the correlation coefficients determined from the known values of two series of known data on the parameters of the earth's rotation separated from the reference data by an interval i and j, and the brackets $\langle\ \rangle$ denote averaging.

Thus, the formula for predicting universal time can be written in the form of a sum of the prediction obtained directly from the harmonic model and the corrections calculated by the other methods described above:

$$dUT1_{pi} = dUT1_{pgi} + \Delta T_{pri} + \Delta T_{si} + \Delta T_{ai},$$

where $dUT1_{pi}$ is the prediction on the ith day; $dUT1_{pgi}$ is the prediction obtained on the basis of an harmonic model of the earth's rotation developed at SNIIM; ΔT_{pri} is the correction to the prediction of $dUT1_{pgi}$ owing to oceanic tides calculated by the method adapted by IERS; ΔT_{si} is the correction determined by the probabilistic-statistical method [6]; and ΔT_{ai} is the correction obtained using the modified autoregression technique developed at SNIIM.

A final prediction of universal time for transmission to the GLONASS spacecraft without loss of accuracy is conveniently written in a compressed form using six coefficients of an approximate expression computed over the interpolation interval, simultaneously including the series of the known data and the predictions:

$$f_A(t) = a_0 + a_1 t + a_2 \sin(\omega_2 t + \varphi_2) + a_3(\omega_3 t + \varphi_3),$$

where $\omega_2 = 2\pi/T_2$ (T_2 = 27.1 days); $\omega_3 = 2\pi/T_3$ (T_3 = 13.66 days); are specified parameters, and a_0, a_1, a_2, a_3, φ_2, φ_3 are the calculated parameters.

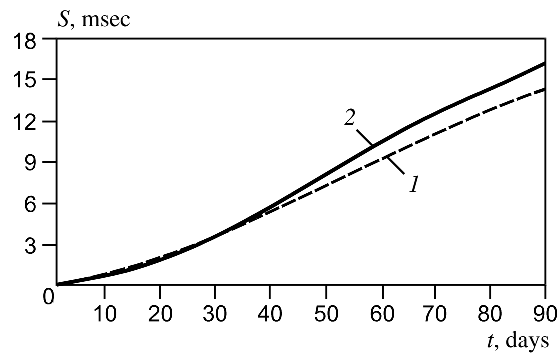

Fig. 2. Comparative estimates of the mean square errors (S) in predictions of *dUT1* for 90 days at SNIIM (curve *1*) and IERS (curve *2*) during the period from January 2008 through May 2009.

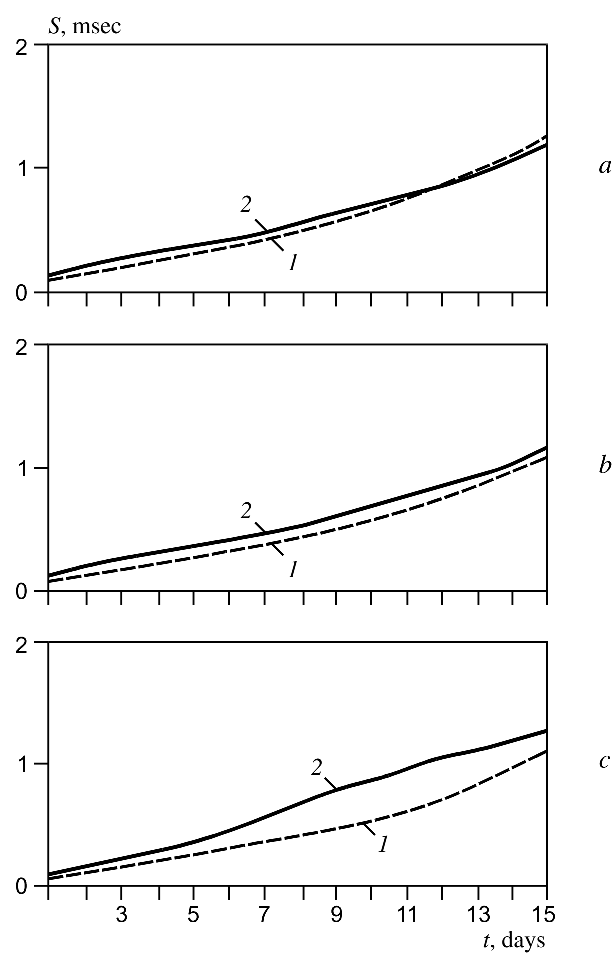

Fig. 3. Comparative estimates of the mean square errors (S) in predictions of *dUT1* for 15 days at SNIIM (curve *1*) and IERS (curve *2*): *a*) during the period from January 2008 through May 2009; *b*) from July 2008 through May 2009; *c*) from January 2009 through May 2009.

Programming for computing the universal time predictions was done in the Fortran-90 (computational modules) and C++ (user interface, the program PVZ) program languages. Depending on the computational regime, the computer time for a single prediction ranges from 20 sec to 2 min.

Representing sudden variations in the speed of the earth's rotation lasting from a year or less to several decades in the form of a sum of harmonic components in order to obtain highly accurate predictions of universal time conflicts with the currently-dominant view that the secular-trend variations are unpredictable. Thus, according to the data of [6], the relative contribution to the total error in universal time predictions per year owing to the instability and unpredictability of the variations in the secular trend is on the order of 99% of the effect on the earth's rotation of the remaining, more regular tidal and seasonal factors. However, as shown in [7] and in this paper, according to independent experts from the State Astronomical Observatory (GAO) of the Russian Academy of Sciences and the 4th Research Institute (4TsNII) of the Ministry of Defense of the Russian Federation, results obtained by SNIIM during 2007–2009 on the prediction of universal time are better than the analogous results obtained by the IERS. This confirms the validity of the proposed approach for taking ten-year variations into account.

It should be noted that the creation of this sort of harmonic model for the earth's rotation, which could be used to obtain highly accurate predictions for a long time without periodic corrections, is extremely difficult for two main reasons: first, because of the limited amount of known *UT1* data (exact since 1962, approximate from 1900, and very approximate since 1650) and, second, because of the impossibility of posing a fully formal statement of the problem. Finally, in any harmonic model it is assumed *a priori* that only regular components are present. At the same time, irregular secular and ten-year variations in the speed of the earth's rotation have a strong influence on the empirically calculated parameters of the harmonic model and, therefore, make it necessary to introduce corrections and refinements for the matching conditions into the model.

For an objective and independent evaluation of the results of the proposed method, predictions calculated at SNIIM twice a week and sent by e-mail to the GAO, the 4TsNII, and the Main Metrological Center of the State Service for Time and Frequency (GSVCh), were compared with the predictions calculated for the same days at the USNO, and of the reference coordinate systems (IERS, USNO).

The results of this comparison for different periods from January 1, 2008, through May 2009 are shown in Figs. 2 and 3. These data show that the accuracy of the prediction using the method developed at SNIIM is higher than that of the IERS predictions (USNO). A similar result was obtained in 2007, which demonstrates the stability advantage of the proposed method for predicting universal time compared to the method used at IERS.

The comparison of plots of the mean square deviations of the predictions in Figs. 2 and 3 indicates a stable positive dynamic in the improvement of the quality of the results obtained in the later periods relative to the entire period of the analysis from January 2008 through May 2009.

The development discussed here is oriented toward improving the ephemeris-time support for GLONASS and, beginning in the middle of 2009, it is planned to use it at the Russian State Service for Time and Frequency for comparing the "universal time" predictions for the daily bulletins in series A and Q. It may also be useful for solving problems of coordinate-time support for various mobile objects which are in places in which signals from global satellite navigation systems are inaccessible for prolonged periods, as well as other applied problems in geophysics and geodynamics.

REFERENCES

1. V. L. Gorshkov, "Prediction techniques in geodynamics," *Izv. GAO*, No. 214, p. 313 (2005).
2. H. Schuh et al., "Earth orientation parameters prediction comparison campaign – first summary," *Geophys. Research Abstr.*, **10**, 46 (2008).
3. Yu. P. Lukashin, Adaptive *Methods for Short-Term Prediction of Time Series* [in Russian], Finansy i Statistika, Moscow (2003), pp. 33–37.
4. Z. M. Malkin and E. A. Skurikhina, "On prediction of EOP," *Communications of IAA*, No. 93, 127 (1996).

5. D. D. McCarthy and G. Petit, *Earth Orientation Parameters Prediction Research: IERS Conventions (2003)*, Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt a.M., *IERS Technical Note No. 32* (2004), p. 144.
6. M. B. Kaufman and D. Yu. Belotserkovskii, "Estimating the accuracy of precomputations of the differences between universal and coordinated times with an advance of up to a year," in: *Studies in Time and Frequency Measurements* [in Russian], *Tr. VNIIFTRI*, Iss. 35(65), 22 (1972).
7. V. M. Tissen, A. S. Tolstikov, and Z. M. Malkin, "Experience with short-term and long-term prediction of parameters of the earth's rotation," in: *Geo-Sibir–2007: Proc. III Int. Congress*, SGGA, Novosibirsk (2007), Vol. 4, part 2, p. 92.