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UT1 PREDICTION BASED ON LONG-TIME SERIES ANALYSIS

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ABSTRACT. A new method is developed for prediction of UT1. The method is based on construction of a general polyharmonic model of the Earth rotation parameters variations using all the data available for the last 80-100 years, and modified autoregression technique. A rigorous comparison of UT1 predictions computed at SNIIM with the prediction computed by IERS (USNO) in 2008-2009 has shown that proposed method provides better accuracy both for ultra-short and long term predictions.

Keywords: Earth rotation, Earth orientation parameters, Universal time (UT1), UT1 prediction, polyharmonic model, autoregression.

1. INTRODUCTION

Evidently, Global Navigation Satellite Systems is the most exacting application about the errors of the forecast of the Earth orientation parameters (EOP). Modern requirements to the EOP predictions accuracy are based on criteria of their negligible influence on the results of positioning and timing, including a possible period of autonomous GNSS satellite functioning. In particular, test computations show that 15-day prediction of *dUT1=UT1-UTC* must be accurate to better than 1 ms, and even more ambitious task is to achieve the accuracy (RMS error) of a 2-month prediction at the level of 2 ms (Pasynkov et al., 2009).

However, in spite of various and sometimes quite sophisticated methods of time series prediction which are used nowadays, the required accuracy is not yet achieved. Improvement of the UT1 prediction accuracy observed during the past years is a consequence of developments in rapid UT1 determination rather than a result of improvement in the forecast technique (Luzum and Nothnagel, 2010). For example, according to the IERS Annual Reports (IERS 2001-2009) and our latest computations, RMS error of 15-day IERS UT1 prediction decreased during the 2000s from 2–3 ms to about 1.3-1.4 ms, and the error of a 2-month prediction remains at a level of 7-11 ms. Further diminution of this value is especially difficult because of peculiarities of the Earth's rotation connected in the first place with sudden unpredictable changes in trend components of the rotation speed. Possible reasons of these phenomena are discussed in many papers, e.g., (Munk and MacDonald, 1960; Shuleykin, 1971, Lambeck, 1980; Sidorenkov, 2002).

This problem could be solved by developing a comprehensive theory of the Earth's rotation taking into account all the acting time-variable forces and torques including their prediction. Unfortunately, at the moment our knowledge of various geophysical processes and their interaction and computational possibilities are too poor to construct such a model with a sufficient accuracy, although this work is under development, see, e.g. (Akulenko, 2002). Therefore we have not yet a physical model of UT1 prediction which can meet the most of practical needs, and development of mathematic tools for this purpose remains to be an actual task.

Currently used methods of highly-accurate EOP forecast are based on statistical analysis of time series of observed EOP estimates. These time series are of finite length, and mostly only a part of input series is used to adjust the prediction model parameters. For *dUT1*, time series segments of the length 2-6 years are usually used. This can be explained by the fact that the models currently used in the EOP services take into account only short-term variations in Earth's rotation speed which are not stable enough at decadal time scale. Such approach can cause the loss of important information on the long-term behavior of the Earth including possible decadal regularities.

To overcome these shortcomings, a new method was developed in the Siberian Metrological Scientific-Research Institute (SNIIM). The method overview and preliminary testing results have been published in (Tissen et al., 2009). In this paper, we provide more details of the method and present new results of its practical application to *dUT*1 prediction.

2. A METHOD PROPOSED TO IMPROVE UT1 PREDICTION

Assessment of the *dUT*1 forecast accuracy, and its dependence on various factors shows that irregularities in long-term trend components make the main contribution to the prediction errors, especially for long-term forecast. It is evident that the impact of error in trend extrapolation grows with the prediction length, and for 1-year forecast trend prediction error gives about 99% of total *dUT*1 prediction error budget. Hence it appears that development of a method of *dUT*1 prediction allowing for trend variations is of great importance for improvement of the accuracy of *dUT*1 prediction.

2.1. Method overview

In our method, *dUT*1 variations for the last 80-100 years are approximated by a polyharmonic models including several tens terms with periods from days to decades. Using such a large reference interval of data together with the specially developed filtering method allows us to reliably separate determined and stochastic component of the *dUT*1 signal, and hence improve its predictability in the future. The computations are made in four following steps.

1. Computation of the *dUT*1 corrections for the solid earth and ocean tides using the IERS Conventions (2003) algorithms (McCarthy and Petit, 2004).

2. Trend modeling applied to the residual series after subtracting the tidal terms from the initial series. See Section 2.2 for details.

3. Computation of seasonal variations of *dUT*1 as the sum of harmonic terms with periods from several days to one year found by the least squares method from the residual series obtained after subtracting all the components found in the previous steps from the initial series. See Section 2.3 for details.

4. Stochastic modeling using modified autoregression method applied to the remaining residual series after subtracting all the components found in the previous steps from the initial series. See Section 2.4 for details.

Finally, we can write the equation for model used as

$$dUT1_{prg,i} = dUT1_{real,0} - dUT1_{pa,0} + dUT_{pa,i} + \Delta T_{pr,i} + \Delta T_{tr,i} + \Delta T_{sz,i} \ , \qquad (1)$$

where

$dUT1_{prg,i}$ is predicted $dUT1$ value at the i-th day in the future;

$dUT1_{real,0}$ is the last observed $dUT1$ value corresponding to the last observed epoch;

$dUT1_{pa,0}$ is predicted $dUT1$ value at the last observed epoch computed from the stochastic (autoregression) model;

$dUT1_{pa,i}$ is predicted $dUT1$ value at the i-th day in the future computed from the stochastic (autoregression) model;

$\Delta T_{pr,i}$, $\Delta T_{tr,i}$, $\Delta T_{sz,i}$ are tidal, trend and seasonal terms correspondingly at the i-th day in the future.

2.2. Trend modeling

As mentioned above, we used a polyharmonic model to approximate the long-term variations in the Earth's rotation. This model is constructed in the following way.

1. The reference interval of at least 65-70 years is selected.

2. The set of test harmonics with periods T_j, $j = 1, \ldots ,N$ with the step ΔT is constructed.

3. For all the harmonics T_j the following operations are performed:

3.1. The reference interval is divided in k overlapping segments of the length T_j with the step $T_j/2$;

3.2. all the segments are superposed in such a way that *dUT1* values at the even segments are taken as is, and *dUT1* values at the odd segments are taken with opposite sign; then the resulting sum will contain the harmonic term with the period T_j multiplied by factor k, while all other terms are multiplied by factor $\sqrt{k}$;

3.3. the amplitude and phase of the harmonic given as $f(t_i) = A_j \sin(2\pi t_i/T_j + \varphi_j)$ are computed by least squares;

3.4. approximation error $\sigma_{anp,j}$ is estimated from the same analysis;

3.5. the criteria of the approximation quality is computed as

$$k_j = \frac{A_j}{\sigma_{anp,j}}. \qquad (2)$$

4. After exhaustion of all the periods T_1–T_N, the term with the maximum value of k_j is selected.

5. The parameters of found harmonics are refined using an analogous strategy with a set of test harmonics given with smaller step in the vicinity of T_j. Such an operation is repeated until the k_j values obtained in two consequent steps differ less than $\varepsilon = 10^{-6}$ (relative error).

6. Found term is subtracted from the time series.

7. Steps 1-6 are repeated until k_j becomes less than given tolerance. This tolerance is determined from the previous analysis of EOP series.

8. Final contribution of the trend component to the $dUT1$ prediction at the i-th day in the future is computed as

$$\Delta T_{tr,i} = \sum_{j=1}^{N_{tr}} A_{tr,j} \sin\left(2\pi t_i / T_{tr,j} + \varphi_{tr,j}\right). \qquad (3)$$

2.3. Seasonal variations

In our method, seasonal correction $\Delta T_{sz,i}$ is computed as the sum of annual, semiannual and other harmonics with periods less than one year. The number of harmonics is not limited beforehand, and their parameters are computed by least squares fit on the interval of 15-25 years. Note that usually only annual and semiannual terms determined on intervals of 4-6 years are considered.

Seasonal terms are computed using the approach used for trend components as described in Section 2.2. Initial set of test harmonics includes terms with periods from 7 days to one year with the step of 1 day. Finally, the contribution of the seasonal component to the $dUT1$ prediction at the i-th day in the future is computed as

$$\Delta T_{sz,i} = \sum_{j=1}^{N_{sz}} A_{sz,j} \sin\left(2\pi t_i / T_{sz,j} + \varphi_{sz,j}\right). \qquad (4)$$

An example of the polyharmonic model including trend and seasonal terms is presented in Table 1.

Table 1. Main terms of the UT1 polyharmonic model.

No.	Period, yr	Amplitude, ms
1	66,00	3000,0
2	33.00	500,0
3	22.00	750,0
4	14.00	160,0
5	10.00	40.0
6	8.00	20.0
7	6.00	30.0
8	4.80	20.0
9	3.60	12.0
10	2.40	12.0
11	2.00	4.0
12	1.70	4.0
13	1.10	2.0
14	1.00	21.0
15	0.90	1.0
16	0.83	1.2
17	0.71	1.1
18	0.58	1.0
19	0.50	8.0
20	0.33	1.0

2.4. Stochastic modeling

For EOP time series stochastic analysis we used a modified autoregression method. This method differs from the standard one by using differences up to the 6th order instead of the original measurements. This allows us to present the model of the analyzed time series as the sum of the reference value x_{op} and decreasing (in magnitude) corrections $x_i^{(n)}$, where n is the order of the difference, while in the standard autoregression technique close values are summarized, which can lead to degradation of the prediction accuracy.

Finally, stochastic (autoregression) part of the prediction in the (i+1)-th step is computed as

$$\begin{aligned} x_{i+1} &= a_0 x_i + a_1 x_{i-1} + a_2 x_{i-2} + \ldots + a_{k-1} x_{i-k} + \ldots \\ &+ b_0 \Delta'_i + b_1 \Delta'_{i-1} + b_2 \Delta'_{i-2} + \ldots + b_{k-2} \Delta'_{i-k+1} + \ldots \\ &+ c_0 \Delta''_i + c_1 \Delta''_{i-1} + c_2 \Delta''_{i-2} + \ldots + c_{k-3} \Delta''_{i-k+2} + \ldots \\ &+ d_0 \Delta'''_i + d_1 \Delta'''_{i-1} + d_2 \Delta'''_{i-2} + \ldots + d_{k-4} \Delta'''_{i-k+3} + \ldots, \end{aligned} \quad (5)$$

where a_j, b_j, c_j, d_j are the autoregression coefficients, $j = 1 \ldots k$); k is the dimension of the autoregression matrix,; $\Delta'_i = x_i - x_{i-1}$ are the first differences; $\Delta''_i = \Delta'_i - \Delta'_{i-1}$ are the second differences; $\Delta'''_i = \Delta''_i - \Delta''_{i-1}$ are the third differences.

3. METHOD REALIZARTION AND TESTING

The prediction method described above was realized in the suite of programs written in Fortran-90 and C++. These programs were used for practical computation of the *dUT*1 predictions. Since January 2008, 90-day predictions were computed about twice a week. The EOP series computed at the IERS Rapid Service/Prediction Centre (IERS RS/PC)[1], USNO was used as reference. In total, during the period from 10 January 2008 till 07 October 2009 156 predictions were computed in real time.

For testing purposes, we compared our prediction results with the IERS RS/PC predictions obtained on the same days using the same reference EOP series. To provide a more detailed comparison, three statistics were computed: RMS error, MAE and maximum error.

Results of processing of the series of predictions described above are presented in Figs. 1 and 2 for 90-day and 7-day respectively. One can see that the method presented in this paper shows better accuracy, especially for shortest (up to about 1 week) and longest (longer about one month) predictions. It should be mentioned that the method details are regularly improved, and one can see that the results obtained for 2009 are substantially better than ones obtained in 2008.

4. CONCLUSION

In this paper a new method of UT1 prediction developed at SNIIM is presented. The main distinction of our method is using of long-time series, up to 100 years, to estimate the trend component of the UT1 series. This trend component is expanded in a polyharmonic time series consisting of 20 and more terms. After removing trend, the residuals are predicted making use of a modified autoregression method.

The method has been rigorously compared with the IERS results using predictions made in 2008-2009 simultaneously at SNIIM and USNO. Method proposed here has shown the better accuracy, and prospects for further improvement are optimistic. Especially important for GNSS applications is an improvement in the short-term prediction with the length of several days. As for the long-term prediction, it was shown that using longtime reference series for trend evaluation can provide better prediction accuracy.

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[1] http://maia.usno.navy.mil/

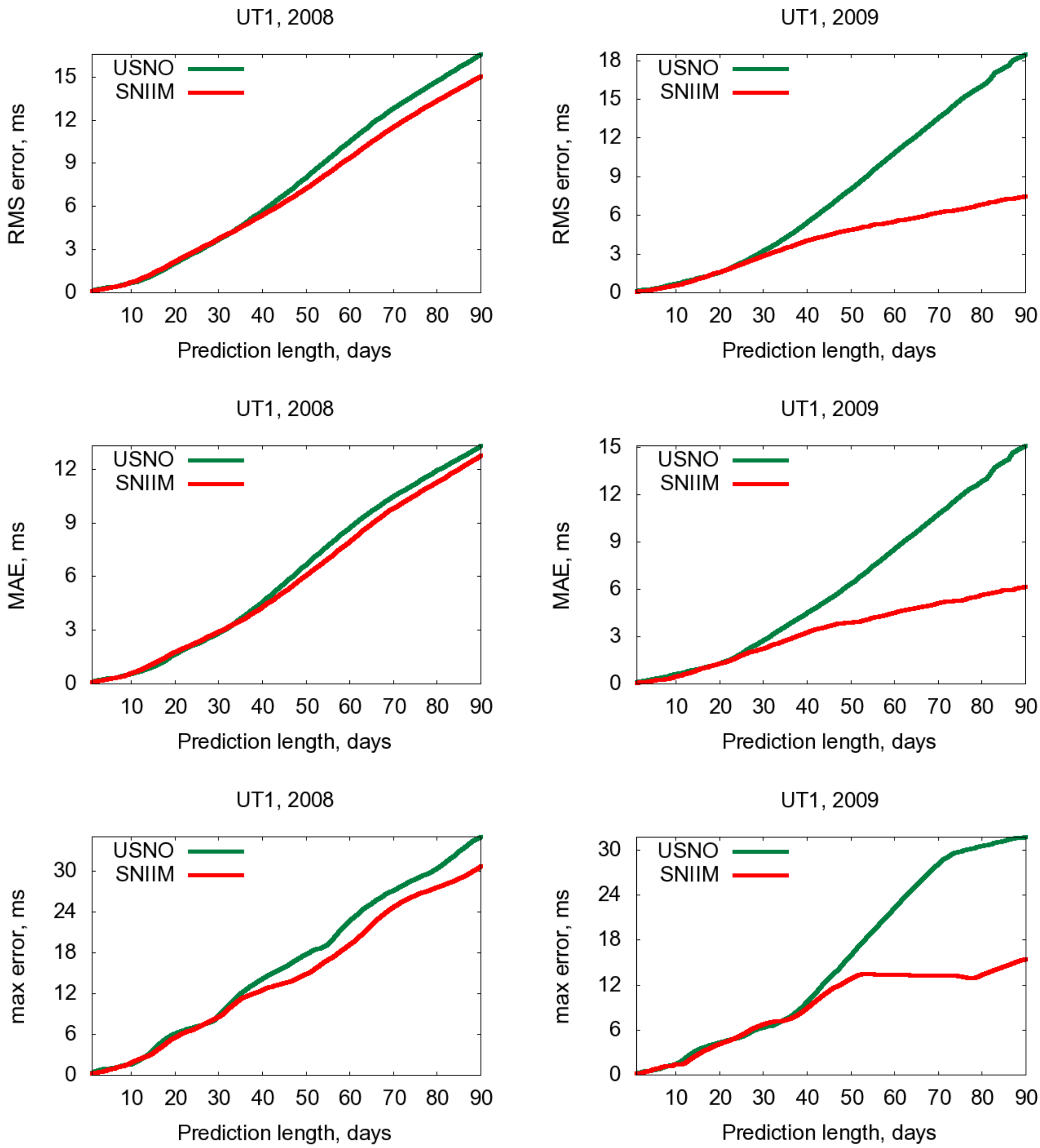

Fig. 1. Errors of UT1 90-day predictions made at SNIIM and USNO in 2008 (left) and 2009 (right). From top to bottom: RMS, MAE and maximum values. Unit: ms.

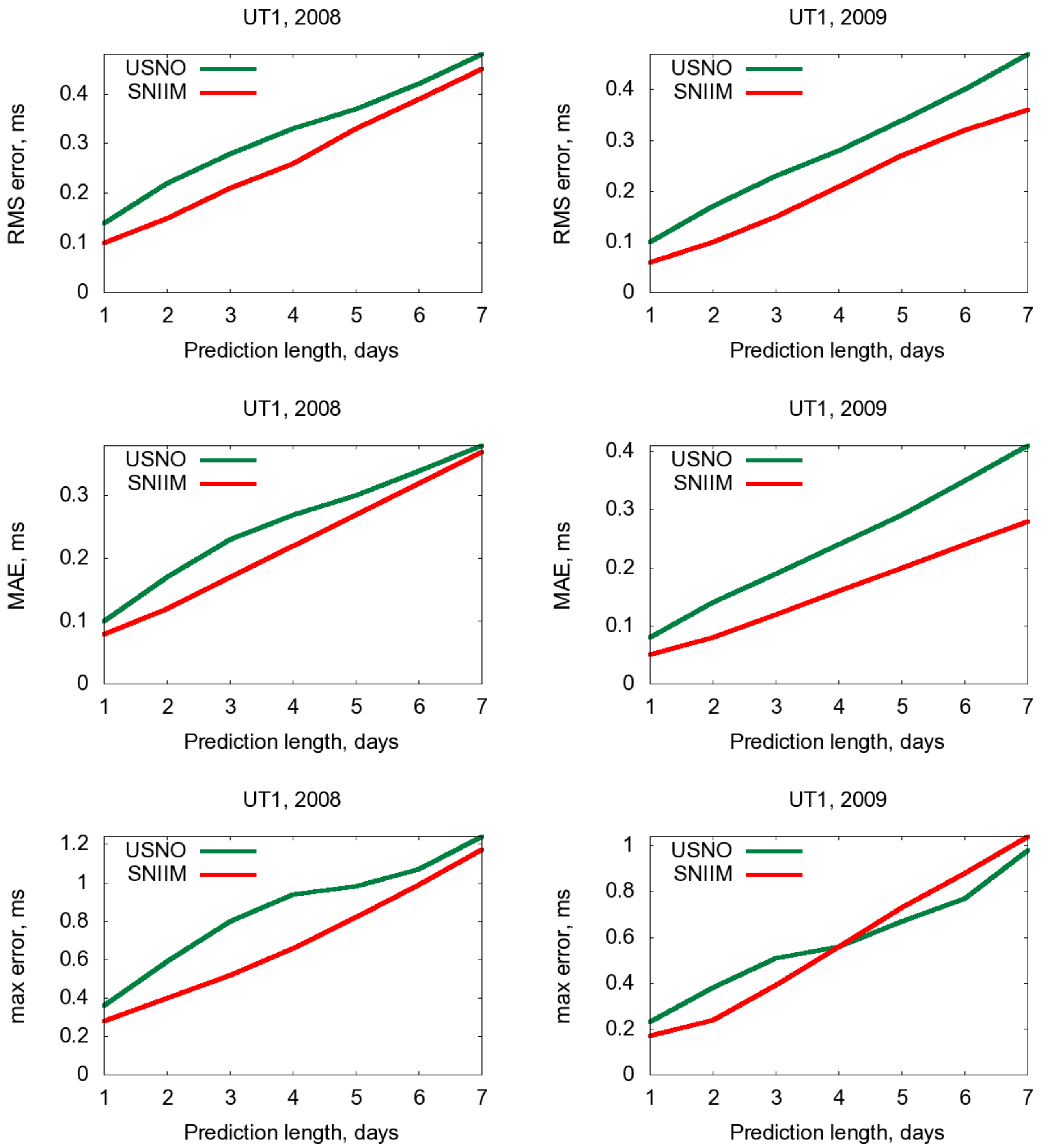

Fig. 1. Errors of UT1 7-day predictions made at SNIIM and USNO in 2008 (left) and 2009 (right). From top to bottom: RMS, MAE and maximum values. Unit: ms.

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