

The current best estimate of the Galactocentric distance of the Sun based on comparison of different statistical techniques

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Abstract

In this paper, the current best estimate of a fundamental Galactic parameter Galactocentric distance of the Sun R_0 has been evaluated using all the available estimates published during the last 20 years. Unlike some other studies, our analysis of these results showed no statistically significant trend in R_0 during this period. However we revealed a statistically valuable improvement in the R_0 uncertainties with time of about 0.2 kpc for 20 years. Several statistical techniques have been used and compared to obtain the most reliable common mean of 52 determinations made in 1992–2011 and its realistic uncertainty. The statistical methods used include unweighted mean, seven variants of the weighted mean, and two variants of median technique. The R_0 estimates obtained with these methods range from 7.91 to 8.06 kpc with uncertainties varying from 0.05 to 0.08 kpc. The final value derived in this analysis is $R_0 = 7.98 \pm 0.15|_{stat} \pm 0.20|_{syst}$ kpc, which can be recommended as the current best estimate of the Galactocentric distance of the Sun. For most of the practical applications the value of $R_0 = 8.0 \pm 0.25$ kpc can be used.

1 Introduction

An accurate knowledge of the distance from the Sun to the center of the Galaxy, R_0 , is important in many fields of astronomy and space sciences such as establishing a distance scale in the Universe, studying the structure and dynamics of the Galaxy, determination of the distance to extragalactic objects, etc. In particular, the primary motivation for this study was a wish to improve the accuracy of modeling of the Galactic aberration (Malkin 2011a).

During decades, many tens of scientifically meaningful determinations of R_0 were made. Various methods and observational data were used to obtain these results, which sometimes substantially differ from each other. The uncertainties associated with this determinations also vary in wide range, and not always correspond to the real accuracy of reported results,

Table 1: Previous average estimates of R_0

| Paper | Period covered | R_0 , kpc |
|---------------------------|----------------|----------------|
| Kerr & Lynden-Bell (1986) | 1974–1986 | 8.5 ± 1.1 |
| Reid (1989) | 1974–1987 | 7.7 ± 0.7 |
| Reid (1993) | 1974–1992 | 8.0 ± 0.5 |
| Nikiforov (2004) | 1974–2003 | 7.9 ± 0.2 |
| Avedisova (2005) | 1992–2005 | 7.8 ± 0.32 |

which was not always properly assessed. For these reasons, it may be difficult for the user to choose the best R_0 value suitable for his application. These shortcomings of the original measurements can be mitigated in an average value or a common mean (CM), which is a generally used procedure. Equally important is to derive a realistic uncertainty associated with this combined value. Both underestimated and overestimated uncertainties are undesirable.

Table 1 shows the latest average estimates of R_0 . All the three values are statistically consistent, i.e. lies inside the overlapping 1σ intervals. However, the uncertainty of the first result is too large to satisfy the need of many exacting applications. The reason may be that Kerr & Lynden-Bell (1986) computed the uncertainty of the final result as the standard deviation (STD) of the individual determinations with respect to the unweighted mean value, and ignored statistical error equal to 0.22. On the other hand, the uncertainty of the results of Nikiforov (2004) and Avedisova (2005) may be underestimated because it seems to be just a formal error coming from the least squares procedure without a check of consistency of the data used in the computation. Notice that the result of Avedisova (2005) is less precise because, most probably, it is obtained from relatively small set of data with respect to the work of Nikiforov (2004). The result of Reid (1993) based on a comprehensive analysis of many data looks as the most realistic both with respect to R_0 value and its uncertainty. However it was obtained about 20 years ago, and evidently needs revision. It should be noted that all these results were obtained with detailed considerations of the physical aspects of observational aspects, but maybe without sufficient statistical analysis of the published estimates. In particular, they used simple (un)weighted mean to compute a statistical estimate of CM without accounting for possible statistical discrepancy of the published data.

A goal of this study is to derive a new best estimate of R_0 . We will use all available determinations published during last 20 years (it will be shown below that earlier results do not contribute substantially to the final result). Besides we will use several statistical techniques which allow processing not only consistent but also discrepant data. This task is not unique. It is well known metrological problem actual for many fields of science and engineering (Cox & Harris 2004). In particular, in this study we consider several methods developed for evaluation of the best estimates of some physical constants (MacMahon et al. 2004; Chen et al. 2011).

This paper is limited in scope to the statistical analysis of the R_0 determinations found in the literature. We will not attempt to discuss the methods involved in the various determinations of R_0 , either theoretical or observational. This study is primarily aimed at testing of a capability of different mathematical approaches in deriving the best current estimates

of the Galactic fundamental constants.

The paper is organized as follows. Section 2 described statistical methods used to evaluate the best estimate of R_0 . Section 3 presents all the available R_0 estimates published during last 20 years and their preliminary analysis. The methods described in Section 2 will be used in Section 4 to obtain the final result. The work is closed with Concluding remarks.

2 Statistical methods used

Let we have n measured values of a physical quantity x_i with associated reported STD (uncertainties) s_i , $i = 1 \dots n$. Our goal is to compute the best estimate of this quantity and its associated uncertainty. This task is known as the problem of computation of a common mean (CM). A solution of the problem is usually not so simple in practice. Input values x_i generally come from various research groups and are obtained with different methods, observational techniques and data sets. Also the uncertainties s_i are, as a rule, obtained not in uniform way, and their statistical properties are usually not known. These circumstances may make input data rather discrepant than consistent, and using simple statistical methods from textbooks on the subject like most often used classical weighted mean may be not justified. For these reason, several alternative statistical methods were developed, see, e.g., MacMahon et al. (2004); Chen et al. (2011); Malkin (2011c) and papers cited therein. For evaluation of R_0 , several of these methods were selected that are most representative and relevant, in our opinion, to this study. They are briefly discussed in this section.

Weighted mean (WM). This classical method is most widely used. The mean value is computed by

$$\bar{x}_w = \frac{\sum_{i=1}^n p_i x_i}{p}, \quad (1)$$

with STD

$$\sigma_{w1} = \frac{1}{\sqrt{p}}, \quad (2)$$

using weights

$$p_i = \frac{1}{s_i^2}, \quad p = \sum_{i=1}^n p_i. \quad (3)$$

The same formulae (1) and (2) are used in computation of CM is also used in the LRW, NR, and MP methods described below. However, these methods use modified procedures to compute the weights of the input measurements.

Another important statistics is the reduced (normalized) χ^2 computed by

$$\frac{\chi^2}{dof} = \frac{\sum_{i=1}^n p_i (x_i - \bar{x}_w)^2}{n - 1}. \quad (4)$$

It is close to unity for consistent measurements, and is much greater than 1 in case of discrepant data or underestimated reported uncertainties.

Least squares approach leads to alternative estimate of the WM uncertainty. The solution of the least square problem $x_i = \bar{x}_w + \varepsilon_i$ with weights p_i gives the same WM value \bar{x}_w , but

another estimate of its uncertainty:

$$\sigma_{w2} = \sqrt{\frac{\sum_{i=1}^n p_i (x_i - \bar{x}_w)^2}{p(n-1)}}. \quad (5)$$

One can see that σ_{w1} , depends on s_i and does not depend on the scatter of x_i , whereas σ_{w2} depends on both relative values of s_i^2 and the scatter of x_i . They are equal to each other in the case of consistent data, i.e. when $\chi^2/dof = 1$. Difference between the two σ estimates may be attributed to systematic errors in x_i and/or underestimating of s_i . It's important to note that neither σ_{w1} nor σ_{w2} use all the input information, namely scatter of the input measurements and absolute and relative values of their uncertainties. After investigation of behavior of these estimates using simulated and real data, Malkin (2001b) proposed the combined estimate:

$$\sigma_{w3} = \sqrt{\sigma_{w1}^2 + \sigma_{w2}^2}. \quad (6)$$

The latter approach can provide more stable and realistic uncertainty estimate in practical cases, see, e.g., (Malkin 2001a). More detailed discussion on these three approaches to compute the WM STD and several simulated and practical examples are given in Malkin (2011c).

Another procedure to compute the WM uncertainty was proposed by Gray et al. (1990). It uses Bayesian approach and is recommended by the authors for evaluation of experimental data in case of doubt about correctness of reported uncertainties. In this method, the CM is also computed by (1), but its STD is given by

$$\sigma_{w4} = \sqrt{\frac{\sum_{i=1}^n p_i (x_i - \bar{x}_w)^2}{p(n-3)}}. \quad (7)$$

One can see that this approach is similar to the least square one, but gives a bit more conservative estimate of the uncertainty.

Unweighted mean (UWM). The UWM estimate can be justified when the uncertainties of the input measurements are not given, not trusted or suspected to be not consistent. This may be also the case for R_0 determinations, see e.g. Kerr & Lynden-Bell (1986). Unweighted CM estimate and its uncertainty is computed by

$$\bar{x}_u = \frac{\sum_{i=1}^n x_i}{n}, \quad (8)$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x}_u)^2}{n(n-1)}}, \quad (9)$$

which corresponds to σ_{w2} computed with unit weights.

Limitation of relative weights (LRW). This procedure is adopted by the Nuclear Data Section of International Atomic Energy Agency (Nichols 2004). It is developed to prevent a measurements with very small reported uncertainty from dominating CM. According to this method, no input value should have a relative weight p_i/p greater than 0.5. When such a value found, its uncertainty should be increased, and then a weighted mean should be recomputed.

Normalized residuals (NR). This method introduced by James et al. (1992) is also intended to limit a relative statistical weights of input values. However, unlike LRW method, the uncertainties of only discrepant data are adjusted. For this purpose, the normalized residuals r_i are computed by

$$r_i = \sqrt{\frac{p p_i}{p - p_i}} (x_i - \bar{x}_w). \quad (10)$$

The limiting value of r_i is defined as

$$r_0 = \sqrt{1.8 \ln n + 2.6}. \quad (11)$$

For any measurement with $|r_i| > r_0$, the uncertainty of the i -th measurement is inflated. The procedure is started from the measurement with the largest $|r_i|$, and repeated until no r_i will be greater than r_0 .

Mandel–Paule (MP). The MP estimate of CM (Paule & Mandel 1982) is defined as x_w computed by (1) with weights

$$w_i = \frac{1}{s_i^2 + s_b^2}, \quad (12)$$

where s_b^2 is computed by solving the equation

$$\sum_{i=1}^n w_i (x_i - \bar{x}_w)^2 = n - 1. \quad (13)$$

This method treats the set of systematic errors of the input measurements as a source of random variability, and s_b^2 is an estimate of the “inter-laboratory” variance. In fact, this method simply increases original uncertainties by an additive variance to make the normalized χ^2/dof equal to unity. So, its use is only justified in a case of the original $\chi^2/dof > 1$. In this respect the MP approach is similar to Malkin (2001b, 2011c). Rukhin & Vangel (1998) defined a modified MP procedure to be as previously with p instead of $(p - 1)$ on the right side of (13). The modified approach produced practically the same result in our case.

Median (M). Another approach routinely used to get the estimate of CM is computation of a median \bar{x}_m . The median is known as a robust statistics less influenced by outliers. However its standard definition does not provide an estimate of error of a median value (it makes it immune to unreliable uncertainties though). A possible approach to compute a median uncertainty can be found in Müller (1995, 2000a). Let \bar{x}_m be the median of x_i , i.e. $\bar{x}_m = med\{x_i\}$. Now we can compute the median of the absolute deviations (MAD) as

$$MAD = med\{|x_i - \bar{x}_m|\}. \quad (14)$$

The uncertainty of \bar{x}_m is then taken as

$$\sigma_m = \frac{1.8582}{\sqrt{n - 1}} MAD. \quad (15)$$

One can see that this estimate of the median uncertainty depends only on the data scatter and not on the input uncertainties. Later Müller (2000b) proposed a method to take account of the uncertainties in input data and thus compute weighted median and its STD. However

its practical realization, as pointed out by the author, is more cumbersome, and the testing results and discussion given therein do not show clear advantage of using weighted median.

Bootstrap median (BM). The bootstrap method is based on a Monte-Carlo procedure to estimate CM and its uncertainty. A random sample (with replacement) of length n is selected from input data, and the median $x_{m,j}$ is computed. After repeating the sampling M times ($j = 1 \dots M$), the best estimate of CM and its STD are computed by

$$\bar{x}_b = \frac{\sum_{j=1}^M x_{m,j}}{M}, \quad (16)$$

$$\sigma_b = \sqrt{\frac{\sum_{j=1}^M (x_{m,j} - \bar{x}_b)^2}{M - 1}}. \quad (17)$$

To obtain a reliable estimates of \bar{x}_b and σ_b , sufficiently large number of sampling should be made. After testing, we found that $M = 10,000$ provides stable result not changing substantially with increasing M .

Note that three methods used in this work, UWM, M, and BM, do not use the uncertainties reported with data.

3 Input R_0 determinations

In this section we strived to collect all the results of determination of R_0 published during last 20 years. This interval was chosen because test computations with earlier data showed no significant difference with final result of this paper presented in Section 4. It will be shown below that the average result converges to a stable value during the period considered in this work. The reader interested in the historical data can refer to, e.g., Kerr & Lynden-Bell (1986); Reid (1993); Surdin (1999); Nikiforov (2004).

All the results found in the literature were used in this work with few exceptions. We did not use Glushkova et al. (1998) later revised by Glushkova et al. (1999), Eisenhauer et al. (2003) revised by Eisenhauer et al. (2005), and Paczyński & Stanek (1998) revised by Stanek et al. (2000).

When both random (statistical) and systematic uncertainties are given, they were summed in quadrature. See Table 2 for details.

If two different values are given for the lower and upper boundaries of the confidence interval (Ghez et al. 2008; Reid et al. 2009b; Vanhollebeke et al. 2009), the mean value of these boundaries was used as the uncertainty of the result. When the authors give several estimates for R_0 without final conclusion, an unweighted average of this estimates was computed. Details are given in Table 3.

The first impression of the input data is a large spread in the reported uncertainties from 0.15 to 1.1 kpc with about 1.5 times improvement during last 20 years (Fig 1). The value of this trend -0.0097 ± 0.0054 kpc/yr is statistically significant. A reason for such large scatter in uncertainty, except the real difference in the accuracy of the methods used, can be the fact that some authors may be not critical enough in estimating the error of the result. Besides, as mentioned above, only few authors give an estimate of systematic error of the reported result in addition to the usually used statistical (formal) error. All these

Table 2: R_0 determinations with estimation of both statistical and systematic errors

| Paper | R_0 , kpc |
|---------------------------|--|
| Nishiyama et al. (2006) | $R_0 = 7.52 \pm 0.10$ _{stat} ± 0.35 _{syst} |
| Groenewegen et al. (2008) | $R_0 = 7.94 \pm 0.37$ _{stat} ± 0.26 _{syst} |
| Trippe et al. (2008) | $R_0 = 8.07 \pm 0.32$ _{stat} ± 0.13 _{syst} |
| Gillessen et al. (2009b) | $R_0 = 8.33 \pm 0.17$ _{stat} ± 0.31 _{syst} |
| Gillessen et al. (2009a) | $R_0 = 8.28 \pm 0.15$ _{stat} ± 0.29 _{syst} |
| Matsunaga et al. (2009) | $R_0 = 8.24 \pm 0.08$ _{stat} ± 0.42 _{syst} |
| Sato et al. (2010) | $R_0 = 8.3 \pm 0.46$ _{stat} ± 1.0 _{syst} |

Table 3: Averaged R_0 values

| Paper | Original R_0 estimates, kpc | |
|--------------------------------|-------------------------------|-----------------|
| Glass et al. (1995) | 8.7 ± 0.7 | 8.9 ± 0.7 |
| Layden et al. (1996) | 7.6 ± 0.4 | 8.3 ± 0.5 |
| Genzel et al. (2000) | 7.9 ± 0.85 | 8.2 ± 0.9 |
| Groenewegen & Blommaert (2005) | 8.6 ± 0.7 | 8.8 ± 0.4 |
| Shen & Zhu (2007) | 7.95 ± 0.62 | 8.25 ± 0.79 |
| Ghez et al. (2008) | 7.96 ± 0.63 | 8.36 ± 0.37 |
| Majaess et al. (2009) | 7.7 ± 0.7 | 7.8 ± 0.6 |

factors can indicate discrepancy between determinations R_0 , which requires careful statistical processing instead of a straightforward computation of a weighted mean. In particular, for this reason, Kerr & Lynden-Bell (1986) proposed to consider unweighted mean value for the best estimate of Galactic constants, including R_0 .

It is important for further analysis to check the absence of the significant systematic drift in input measurements. Many authors, e.g., Reid (1993); Surdin (1999); Nikiforov (2004) and Foster & Cooper (2010) discussed the long-term trends in R_0 estimates. Reid (1993); Surdin (1999) and Nikiforov (2004) showed the large negative slope in the 1970s to the beginning of the 1990s, As to the last 20 years, Surdin (1999) showed practically no drift for 1990–1998, Nikiforov (2004) showed zero or maybe small positive drift for 1990–2003, and Foster & Cooper (2010) revealed the large positive slope for 1992-2010.

To clarify this contradiction we repeated the computation of the trend in R_0 estimates during the last 20 years using the full data set considered in this paper. All the values listed in Table 4 are depicted in Fig. 2 with the epochs computed with a fraction of the year based on the publication date taken from ADS or journal number. When the month is not specified, the year fraction 0.5 was accepted. In a few cases of coincident epochs, a small fraction of a year was voluntarily added or subtracted. From processing of this data we obtained the value -0.011 ± 0.011 kpc/yr for weighted slope and $+0.006 \pm 0.010$ kpc/yr for unweighted one.

Comparing these data with Foster & Cooper (2010) we can conclude that our results do not confirm the large trend in the R_0 estimates observed by the authors. Our computations also revealed a linear trend but much smaller and statistically unreliable, which corresponds to the results of Surdin (1999) and Nikiforov (2004).

Another simple but effective test of the significance of the long-term trend in the data

Table 4: R_0 estimates used in this study

| R_0 , kpc | STD | Reference |
|-------------|------|--------------------------------|
| 7.9 | 0.8 | Merrifield (1992) |
| 8.1 | 1.1 | Gwinn et al. (1992) |
| 7.6 | 0.6 | Moran et al. (1993) |
| 7.6 | 0.4 | Maciel (1993) |
| 8.09 | 0.3 | Pont et al. (1994) |
| 7.5 | 1.0 | Nikiforov & Petrovskaya (1994) |
| 7.0 | 0.5 | Rastorguev et al. (1994) |
| 8.8 | 0.5 | Glass et al. (1995) |
| 7.1 | 0.5 | Dambis et al. (1995) |
| 8.3 | 1.0 | Carney et al. (1995) |
| 8.21 | 0.98 | Huterer et al. (1995) |
| 7.95 | 0.4 | Layden et al. (1996) |
| 7.55 | 0.7 | Honma & Sofue (1996) |
| 8.1 | 0.4 | Feast (1997) |
| 8.5 | 0.5 | Feast & Whitelock (1997) |
| 7.66 | 0.54 | Metzger et al. (1998) |
| 8.1 | 0.15 | Udalski (1998) |
| 7.1 | 0.4 | Olling & Merrifield (1998) |
| 8.51 | 0.29 | Feast et al. (1998) |
| 8.2 | 0.21 | Stanek & Garnavich (1998) |
| 8.6 | 1.0 | Surdin (1999) |
| 7.4 | 0.3 | Glushkova et al. (1999) |
| 7.9 | 0.3 | McNamara et al. (2000) |
| 8.67 | 0.4 | Stanek et al. (2000) |
| 8.2 | 0.7 | Nikiforov (2000) |
| 8.24 | 0.42 | Alves (2000) |
| 8.05 | 0.6 | Genzel et al. (2000) |
| 8.3 | 0.3 | Gerasimenko (2004) |
| 7.7 | 0.15 | Babusiaux & Gilmore (2005) |
| 8.01 | 0.44 | Avedisova (2005) |
| 7.62 | 0.32 | Eisenhauer et al. (2005) |
| 8.7 | 0.6 | Groenewegen & Blommaert (2005) |
| 7.2 | 0.3 | Bica et al. (2006) |
| 7.52 | 0.36 | Nishiyama et al. (2006) |
| 8.1 | 0.7 | Shen & Zhu (2007) |
| 7.4 | 0.3 | Bobylev et al. (2007) |
| 7.94 | 0.45 | Groenewegen et al. (2008) |
| 8.07 | 0.35 | Trippe et al. (2008) |
| 8.16 | 0.5 | Ghez et al. (2008) |
| 8.33 | 0.35 | Gillessen et al. (2009b) |
| 8.7 | 0.5 | Vanhollebeke et al. (2009) |
| 7.58 | 0.40 | Dambis (2009) |
| 7.2 | 0.3 | Bonatto et al. (2009) |
| 8.4 | 0.6 | Reid et al. (2009a) |
| 7.75 | 0.5 | Majaess et al. (2009) |
| 7.9 | 0.75 | Reid et al. (2009b) |
| 8.24 | 0.43 | Matsunaga et al. (2009) |
| 8.28 | 0.33 | Gillessen et al. (2009a) |
| 7.7 | 0.4 | Dambis (2010) |
| 8.1 | 0.6 | Majaess (2010) |
| 8.3 | 1.1 | Sato et al. (2010) |
| 7.80 | 0.26 | Ando et al. (2011) |

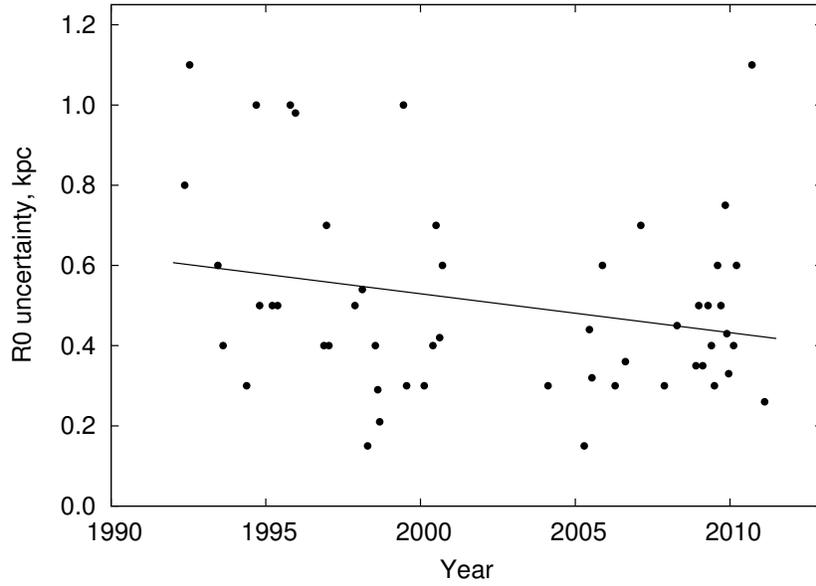


Figure 1: Uncertainty of R_0 estimates used in this study. Unit: kpc.

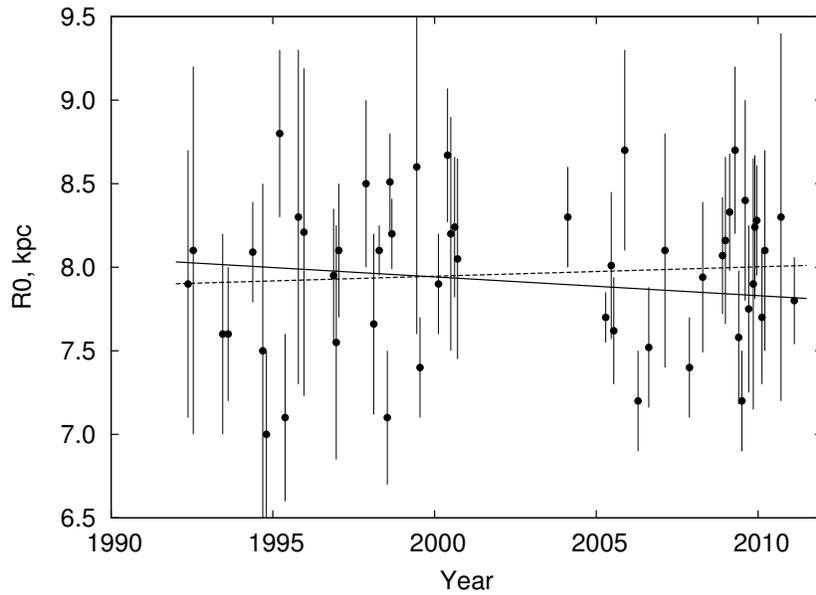


Figure 2: R_0 estimates used in this study. The weighted (solid line) and unweighted (dashed line) trend is also shown. Unit: kpc.

Table 5: Average estimates of R_0 obtained by different statistical methods. The second and third columns contain references to formulas used for computation of the result given in the last column. See Section 2 for method designations

| Method | R_0 , kpc | χ^2/dof |
|-----------------------|-------------------|--------------|
| WM with σ_{w1} | 7.909 ± 0.051 | 1.227 |
| WM with σ_{w2} | 7.909 ± 0.056 | |
| WM with σ_{w3} | 7.909 ± 0.076 | |
| WM with σ_{w4} | 7.909 ± 0.058 | |
| UWM | 7.960 ± 0.062 | |
| LRW | 7.909 ± 0.051 | 1.227 |
| NR | 7.909 ± 0.051 | 1.227 |
| MP | 7.911 ± 0.059 | 1.000 |
| M | 8.060 ± 0.075 | |
| BM | 8.028 ± 0.083 | |

(not necessarily linear) is the Abbe criterion. Strictly speaking, it is intended to test the null hypothesis that all the measurements have the same mathematical expectation. The Abbe statistics can be computed as the ratio between the Allan variance AVAR (see details of this statistics in Malkin (2008, 2011b)) and the variance of the measurements VAR:

$$\begin{aligned}
 q &= \frac{AVAR}{VAR}, \\
 AVAR &= \frac{\sum_{i=1}^{n-1} (y_i - y_{i+1})^2}{2(n-1)}, \\
 VAR &= \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1},
 \end{aligned} \tag{18}$$

where the mean value \bar{x} is computed by (8).

In presence of significant trend, one can expect that VAR will be substantially greater than AVAR. In other words, if the value of q is less than corresponding percentage point, we have to reject the null hypothesis, i.e. confess that the input data contain systematic shift. In our case, we have obtained $q = 1.26$, which is well greater than the 1% critical value 0.69.

Finally, we can conclude that no statistically significant trend is found in the analyzed data during last 20 years.

4 Evaluation of R_0

The results R_0 determination in 1992–2011 listed in Table 4 were processed with 10 statistical techniques described in Section 2. The final results obtained by all the methods are presented in Table 5.

For MP estimate, we also have got an estimate of the inter-laboratory error s_b , which is equal to 0.16 kpc. It shows a level of consistency between the input R_0 determinations. Also, this value was added in quadrature to the original uncertainties according to (12) to derive the final MP estimate.

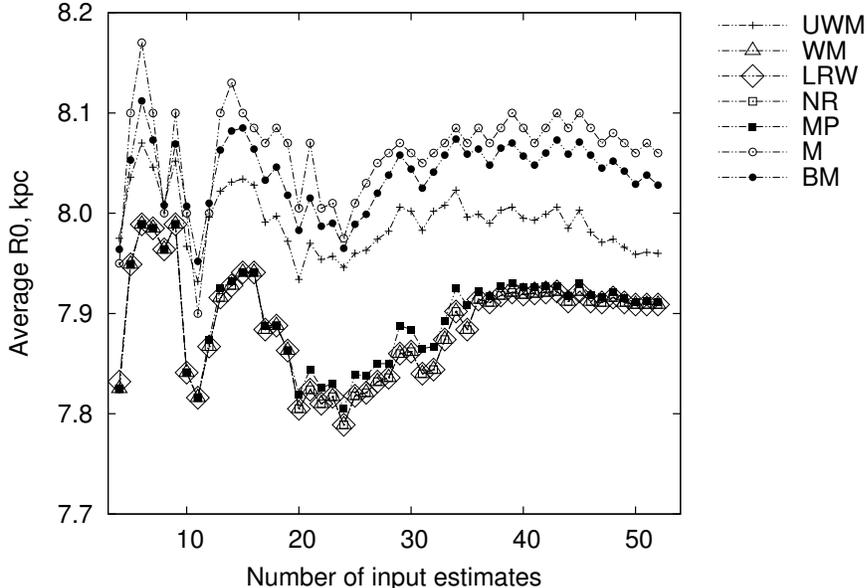


Figure 3: Convergence of R_0 estimates. The abscissa axis shows the number of input estimates counted back from the latest one. Unit: kpc.

It should be noted that the uncertainty of the LRW, NR, and MP estimates originally computed as σ_{w1} can also be computed as σ_{w2} , σ_{w3} or σ_{w4} . We made corresponding calculations, but in our case the results are identical to the WM, and are not presented here. For more discrepant data these modifications may worth consideration.

Figure 3 shows the convergence of the results obtained with different statistical methods with respect to the number of input data used in computation of CM. The data sets used for this computation always begins with the last determination and have different extents in the past. This test is intended to estimate how much historical data contribute to the current best estimate. From this test, we can conclude that only the last 35-40 determinations made during the last about 15 years practically define the current best estimate of R_0 .

There is no well-founded choice between results obtained with different statistical techniques. For this reason, it seems reasonable to accept the final R_0 value as the middle of the interval overlapping all the estimates of CM with their 1σ uncertainties, namely $[7.909 - 0.076, 8.060 + 0.075] = [7.833, 8.135]$. A half of the span of this interval can be accepted as the uncertainty of the final R_0 estimate. Thus, the following final value of the Galactocentric distance can be accepted:

$$R_0 = 7.98 \pm 0.15 \text{ kpc.} \quad (19)$$

In the framework of the classical method, the value (19) considered as a confidence interval corresponds to the confidence probability of 0.995. However this value of uncertainty is only statistical one indeed. Various considerations on possible sources and values of systematic errors in determinations of R_0 can be found in Nishiyama et al. (2006); Kerr & Lynden-Bell (1986); Reid (1993); Nikiforov (2004); Groenewegen et al. (2008); Nikiforov (2008); Trippe et al. (2008); Gillessen et al. (2009a); Matsunaga et al. (2009); Sato et al. (2010). Analysis of the systematic errors of the published R_0 estimates is beyond the scope

of this study. However, it should be mentioned that results of test computations with the original and recomputed R_0 estimates suggested by Nikiforov (2004, 2008) did not show any significant impact on the final value.

Taking into account that the R_0 measurements analyzed in this study were derived from observations of different objects and making use of several different methods with different systematic errors, we can presume that these errors are generally independent between methods, and hence are substantially mitigated in the averaged value (19). Perhaps 0.15–0.2 kpc is a realistic estimate for remaining systematic error in the average result. On the other hand, a bandwagon effect in the R_0 measurements is suspected by many authors, e.g., Reid (1993); Nikiforov (2004) to be substantial. It may bias the current best estimate, but there is no way to quantify this effect.

Finally, the following value of the Galactocentric distance of the Sun has been obtained in this study:

$$\mathbf{R}_0 = 7.98 \pm 0.15 |_{\text{stat}} \pm 0.20 |_{\text{syst}} \text{ kpc.} \quad (20)$$

which can be considered as the current best estimate of R_0 . For most of the practical applications the value of $R_0 = 8.0 \pm 0.25$ kpc can be sufficient.

5 Concluding remarks

In this paper we present a new current best estimate of the distance from the Sun to the center of the Galaxy, R_0 , derived from comparison of estimates obtained with different statistical techniques. Some techniques gives mostly “theoretical” estimates without accounting for possible discrepancy in the data used. Other were developed just to evaluate discrepant data and thus should give more realistic estimates.

In spite of R_0 estimates used in this study were obtained by different methods and object samples, our analysis has shown that they are rather consistent than discrepant. From the results given in Table 5, one can see that the same or very close values were obtained with WM, LRW, NP, and MP methods. This is an evidence that there are no large systematic errors in the estimates available in literature. However, although the estimates derived by several variants of the weighted mean approach give close values of R_0 , their uncertainties differ substantially ranging from 0.51 to 0.76 kpc. Besides, the estimates derived by two variants of median-family technique differ substantially from the WM-family results. All this means that R_0 determinations made during the last 20 years are not fully consistent yet, and using and comparison of different statistical techniques is justified to obtain a reliable combined R_0 estimate. The same can be said about other astronomical constants derived from combined processing of different determinations.

It is important that our R_0 estimate is quite consistent with the values found by Reid (1993) (8.0 ± 0.5 kpc), Nikiforov (2004) (7.9 ± 0.2 kpc), and Avedisova (2005) (7.8 ± 0.32 kpc). We can conclude from this that the R_0 value is to the moment well established around 8.0 kpc with an error of a few tenths of kpc. This fact, together with absence of a significant trend in the R_0 determination made during last 20 years, can lead to a conclusion that the bandwagon effect may be not so significant as suspected.

Finally, the fast convergence of the R_0 determinations found in this study open a possibility of improving R_0 uncertainty substantially during coming years. It is interesting to notice

that the results listed in Table 1 also show a strong tendency to decreasing the R_0 error with time. Unfortunately, it is mainly explained not by improvement of the observational results, but rather by inhomogeneity of the methods used to estimate the uncertainty of the average value, as discussed above.

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