Using Modified Allan Variance for Time Series Analysis

39

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Abstract

Allan Variance (AVAR) was introduced more than 40 years ago as an estimator of the stability of frequency standards. Now it is also used for investigations of time series in astronomy and geodesy. However, there are several issues with this method that need special consideration. First, unlike frequency measurements, astronomical and geodetic time series usually consist of data points with unequal uncertainties. Thus one needs to apply data weighting during statistical analysis. Second, some sets of scalar time series naturally form multidimensional vector series. For example, Cartesian station coordinates form the 3D station position vector. The original AVAR definition does not allow one to process unevenly weighted and/or multidimensional data. To overcome these deficiencies, AVAR modifications were proposed in Malkin (2008. On the accuracy assessment of celestial reference frame realizations. J Geodesy 82: 325–329). In this paper, we give some examples of processing geodetic and astrometric time series using the classical and the modified AVAR approaches, and compare the results.

Keywords

Allan variance • Time series analysis

1 Introduction

The scatter of a geodetic time series provides a good measure of the series quality (and its more explicit statistical characteristics). One of the most effective approaches to analyze the noise component of a measured signal is the Allan Variance (AVAR) originally developed for the evaluation of the stability of time and frequency standards (Allan 1966). In particular, its advantage is the weak dependence of the noise parameter estimate on low-frequency components of the signal under study.

AVAR has already had a rather long history in geodesy and astrometry. It was used to study station coordinates time series (Malkin and Voinov 2001; Le Bail and Feissel-Vernier 2003; Le Bail 2006; Feissel-Vernier et al. 2007), radio source position stability (Feissel et al. 2000; Gontier et al. 2001; Feissel-Vernier 2003), the Earth orientation parameters (EOP) series (Gambis 2002), geocenter motion (Feissel-Vernier et al. 2006). AVAR can be also used to analyze the spectral characteristics of the signal (Feissel et al. 2000; Feissel-Vernier 2003; Feissel-Vernier et al. 2006, 2007) and Hurst parameter (Bregni and Primerano 2005). A detailed review and history of using AVAR in astrometry and geodesy can be found in Malkin (2011).

However, application of AVAR to analysis of the time series of geodetic and astrometric measurements yields sometimes unsatisfactory results because the analyzed series consists of the data points with unequal uncertainties. This necessitates proper weighting not provided by the original AVAR definition. Besides, we often deal with multidimensional values, e.g., terrestrial coordinates and/or velocities (3D or 6D), celestial coordinates and/or proper motions (2D or 4D). To provide adequate analysis of such kind of data,

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AVAR modifications WAVAR, MAVAR, and WMAVAR have been proposed by Malkin (2008a). They are explained in Sect. 2.

In Sect. 3, several practical examples are given to show how proposed AVAR modifications can be applied to investigation of real geodetic and astrometric data. These examples also allow us to better understand some practical features of various types of AVAR estimates.

We do not consider here the applications of AVAR to spectral or persistency analysis. Such applications require computation of modified AVAR on different averaging intervals, which should be straightforward.

2 AVAR and Its Modifications

In this section, we give definitions of AVAR modifications proposed in Malkin (2008a) to analyze real time series. The classical AVAR is defined as

$$\sigma^2 = \frac{1}{2(n-1)} \sum_{i=1}^{n-1} (y_i - y_{i+1})^2 \,. \qquad (39.1)$$

where y_i are measurements, $i = 1,\ldots,n$.

This definition is not satisfactory for the analysis of real measurements, which generally have different uncertainties and thus should be unevenly weighted during analysis to obtain realistic and statistically meaningful estimates for the investigated parameters. For such kind of data, the weighted AVAR, WAVAR, is introduced in Malkin (2008a). It can be defined as

$$\sigma_1^2 = \frac{1}{2p} \sum_{i=1}^{n-1} p_i\,(y_i - y_{i+1})^2, \qquad p = \sum_{i=1}^{n-1} p_i, \qquad p_i = (s_i^2 + s_{i+1}^2)^{-1} \,. \qquad (39.2)$$

where s_i are the measurement uncertainties.

Another modification of AVAR proposed by Malkin (2008a) is intended for processing of multidimensional data. The latter can be considered as a k-dimensional vector $y_i = \left(y_i^1,\ y_i^2,\ \ldots,\ y_i^k\right),\ i = 1,\ \ldots,\ n$, with standard errors $s_i = \left(s_i^1,\ s_{i,}^2\ \ldots,\ s_i^k\right)$. The corresponding weighted multidimensional AVAR, WMAVAR, is given by

$$\sigma_2^2 = \frac{1}{2p} \sum_{i=1}^{n-1} p_i\, d_i^2, \qquad d_i = \left|y_i - y_{i+1}\right|, \qquad p = \sum_{i=1}^{n-1} p_i \,. \qquad (39.3)$$

Strictly speaking, the weights p_i should be computed in accordance with the error propagation law.

$$p_i = \left(\sum_{j=1}^{k} \left\{ \left[\left(y_i^j - y_{i+1}^j\right)/\, d_i\right]^2 \left[\left(s_i^j\right)^2 + \left(s_{i+1}^j\right)^2\right] \right\}\right)^{-1} . \qquad (39.4)$$

However, the expression Eq. 39.4 has a singular point in the case of $di = 0$, i.e. of equal adjacent measurements. Hence, this case require special treatment, e.g. assigning unit weight for close (near-)zero value of di. To avoid the singularity, the following simplified formula for p_i is recommended in Malkin (2008a) for practical use:

$$p_i = \left(\sum_{j=1}^{k} \left[\left(s_i^j\right)^2 + \left(s_{i+1}^j\right)^2\right]\right)^{-1} . \qquad (39.5)$$

No significant difference between results computed with weights given by Eqs. 39.4 and 39.5 was found during real data processing.

In the case of $p_i = 1$, we have unweighted multidimensional AVAR (MAVAR). It is easy to see that WMAVAR is a universal definition which includes all others as special cases. It can be also noted that for homogeneous time series with close s_i, MAVAR (WMAVAR) values should be equal to AVAR (MAVAR) ones computed for one of k vector dimensions and multiplied by √k.

In noise component analysis, one often uses Allan Deviation ADEV computed as the square root of AVAR. Correspondingly, ADEV modifications, WADEV, MADEV, and WMADEV can be used in most practical applications, including this study.

3 Practical Examples

In this section, we give several examples of using modified ADEV. Four examples will be considered: comparison of station displacement time series, investigation of source position stability, comparison of celestial pole offset (CPO) time series, and quality assessment of celestial reference frame (CRF) realization (radio source position catalogues).

3.1 Station Position Series

ADEV is an effective tool for investigation of noise characteristics of station position time series (Malkin and Voinov 2001; Le Bail and Feissel-Vernier 2003; Le Bail 2006; Feissel-Vernier et al. 2007). However, the classical ADEV applied to a series with heterogeneous

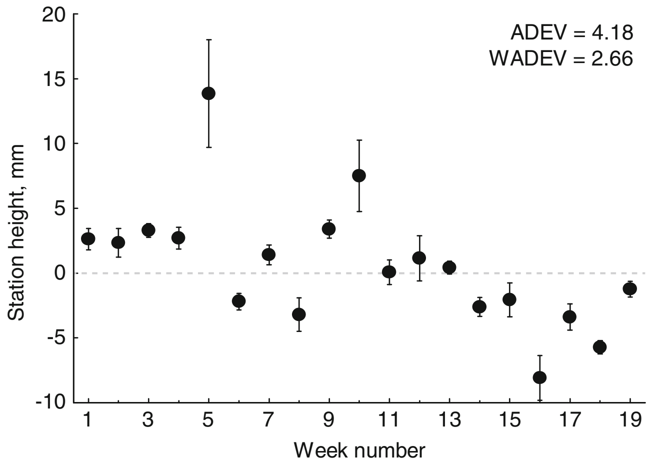

Fig. 39.1 Difference between ADEV and WADEV in case of station height time series with implied outliers

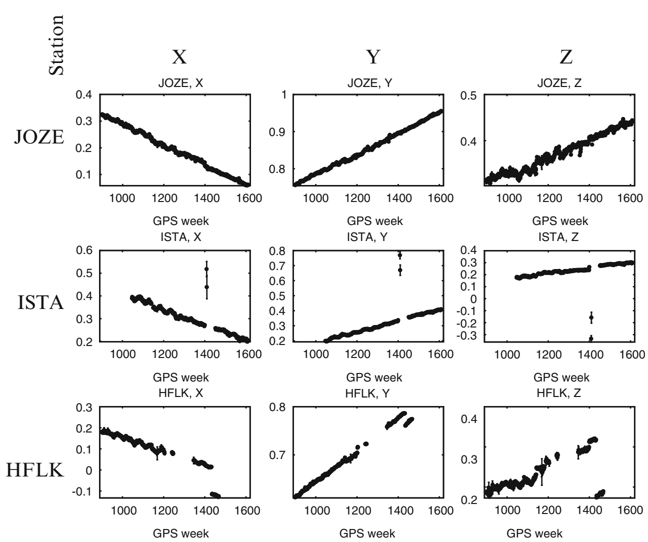

Fig. 39.2 EPN station position time series. X, Y, and Z are the station Cartesian geocentric coordinates. Unit: m

uncertainty can give a biased estimate of the actual signal characteristics.

As the first example, let us consider a series of weekly station height estimates depicted in Fig. 39.1. One can see that one to three points may be considered as excludable because of large bias. Their inclusion in detailed analysis (e.g. scatter statistics) depends on the method used to detect outliers. Using WADEV mitigates the impact of implied outliers without application of special statistics.

Now, let us consider three station position time series provided by the European Permanent GPS Network (EPN) Central Bureau.[1] These series are shown in Fig. 39.2, and

[1] http://epncb.oma.be/

Table 39.1 Scatter characteristics for EPN station position time series shown in Fig. 39.2. Unit: mm

	ADEV			WADEV			
Station	X	Y	Z	X	Y	Z	3D WADEV
JOZE	2.9	1.1	2.7	1.6	1.0	2.0	2.8
ISTA	10.2	17.1	24.0	2.3	1.5	1.9	3.3
HFLK	5.2	1.9	5.6	20.2	2.9	21.2	28.9

scatter statistics is given in Table 39.1. Station JOZE shows stable behavior without jumps and outliers; for this station all the estimates give close results. Station ISTA has two outliers of several decimeters with large uncertainties; for this station using unweighted estimate gives unsatisfactory result, and using WAVAR allows us to practically eliminate outliers. The last case of station HFLK shows in contrast to the previous example unsatisfactory result obtained with WAVAR.

The reason is that the HFLK position uncertainties were relatively large in the period when the station showed position stability and became much smaller to the end of the series where the position jump occurred. As a result, the position estimates around the jump epoch were entered to the statistics with large weight.

3.2 Source Position Series

Investigation of the noise characteristics of source position time series are usually aimed at ranking of sources by time series statistics, and compiling list of sources that are not stable enough to be solved in VLBI global solution, and require special handling. In particular, computation of some quantitative source stability indices are desirable to make a selection of the International Celestial Reference Frame (ICRF) defining sources as objective as possible (Ma et al. 1998; Feissel-Vernier 2003; Gordon et al. 2008; Malkin 2008b).

In Malkin (2009) we investigated source position time series submitted by VLBI analysis centers in the framework of preparation of the Second ICRF realization ICRF2 (Ma et al. 2009). These series were computed using different software and/or analysis strategies, which makes their comparison especially interesting and instructive. Each analyzed series consists of source positions obtained from analysis of 24-h observing sessions (one source position estimate for each session).

Out of several scatter indices generally used for analysis of source position time series we used the following two: weighted root-mean-square (WRMS) residuals of the session source position with respect to weighted mean position (the weights are inversely proportional to the square of the reported position uncertainty), and WADEV. Our study

(Malkin 2009) confirmed that both statistics have their own advantages and disadvantages. WRMS estimate is affected by trend-like and low-frequency signal components.

In contrast, WADEV does not depend on the slow position variations. However, it may give inadequate estimate of the scatter index in the case when the time series contains jumps but is stable between jumps. In order to get a more general measure of source position stability, a composite index of WRMS and WADEV can be used.

It can be noted that some examples given above may seem to be artificial. It is common practice to identify and reject outliers before further statistical analysis. However, as already mentioned above, it can be not a trivial in all the time series considered here, and others we meet in our work. So, our intention is to show that weighted ADEV (WADEV) provides more robust estimate in case of outliers, provided the outliers have large uncertainties, which is quite common, see e.g. Figs. 39.1 and 39.3 (next section). In such a case, the result obtained with WADEV is more independent of the quality of outlier detection procedure used.

3.3 CPO Series Comparison

Investigation of the noise characteristics of CPO time series is important for different tasks, such as quality assessment of EOP series and weighting EOP series during combination (Gambis 2002). Here we consider ten series of the CPO coordinates *dX* and *dY* computed at International VLBI Service for Geodesy and Astrometry (IVS) analysis centers,[2] including the IVS combined series. They are depicted in Fig. 39.3 in a deliberately small scale to show most of the outliers.

A visual inspection of Fig. 39.3 reveals, for example, that OPA series seems to be the noisiest, and IAA series seems to be practically noiseless. However, a deeper look at the data shows that the first impression is wrong. First of all, the series contain different number of sessions. One also notices that most of the outliers have enormously large uncertainties. Both factors should be taken into account. In particular, it requires to compute weighted scatter index and compare subsets of original series consisted of the sessions common for all compared CPO series.

In Tables 39.2 and 39.3, the results of computation of several statistics are presented. The former table shows the impact of data weighting only. The latter table provides scatter indices computed using the same data set for all the series, i.e. free of selection effect. The statistics used are WRMS, MADEV (unweighted 2D ADEV estimate) and WMADEV (weighted 2D ADEV estimate). WRMS values given in the table are computed as average of those for dX and dY. In fact, WRMS1 is just the WRMS value of the reported CPO estimates. Every statistics is computed in two variants: for

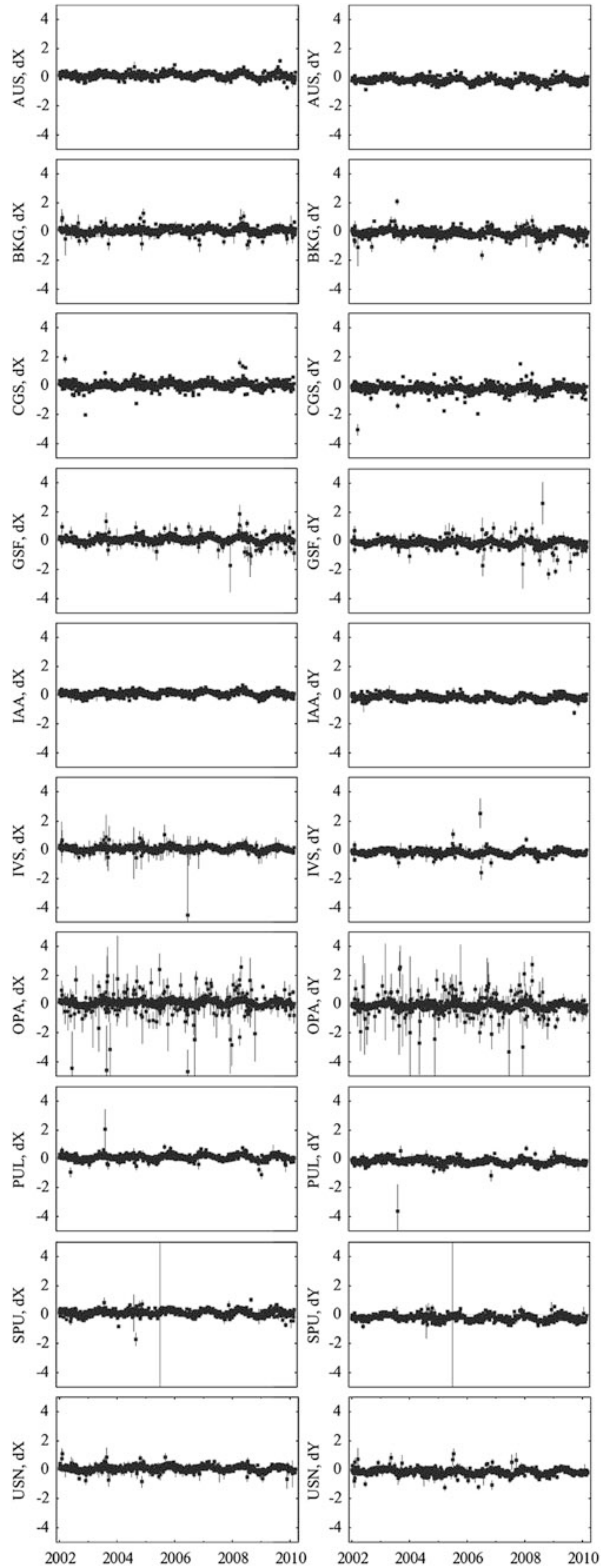

Fig. 39.3 VLBI CPO time series. Units: mas

Table 39.2 Statistics of the VLBI CPO time series depicted in Fig. 39.3. The statistics indexed with 1 are computed from raw data, the statistics indexed with 2 are computed after removing CPO model and linear trend. Units: µas

Series	No. of sessions	WRMS1	WRMS2	MADEV1	MADEV2	WMADEV1	WMADEV2
AUS	862	251	113	206	206	177	177
BKG	1212	191	101	245	245	163	163
CGS	1083	224	129	301	300	200	200
GSF	1276	197	86	279	279	137	137
IAA	1101	211	97	173	173	146	146
IVS	1158	202	80	236	236	125	126
OPA	1437	189	87	680	680	149	148
PUL	1167	213	87	215	215	137	137
SPU	854	253	115	227	227	180	180
USN	1208	195	86	200	200	132	132

Table 39.3 The same as Table 39.2, but computed for common sessions

Series	No of sessions	WRMS1	WRMS2	MADEV1	MADEV2	WMADEV1	WMADEV2
AUS	740	251	113	195	195	176	176
BKG	740	197	101	180	180	159	158
CGS	740	226	128	222	221	203	203
GSF	740	199	82	145	145	131	130
IAA	740	214	98	165	165	151	151
IVS	740	209	79	139	139	125	125
OPA	740	189	78	141	141	128	127
PUL	740	217	90	152	152	139	139
SPU	740	254	114	223	223	179	179
USN	740	192	85	146	146	132	132

Table 39.4 Statistics of the two CPO time series computed with two source catalogues ICRF1 and ICRF2. Unit: µas

CRF	WADEV(dX)	WADEV(dY)	WMADEV
ICRF1	102	107	149
ICRF2	92	90	129

original CPO estimates as reported by authors (indexed with 1), and for values corrected for CPO model ZM2 (Malkin 2007) and linear trend (indexed with 2). A detailed discussion of the results of these computations is beyond the scope of this paper. Let us just mention several conclusions.

– The comparison of "1" and "2" variants confirms that WRMS depends heavily on the model of systematic errors, whereas WADEV and WMADEV do not.
– Using weighted ADEV estimates allows us to severely mitigate the influence of outliers. It is especially visible for the OPA series which includes many CPO estimates having large bias and uncertainty (coming from sessions with poor geometry), evidently for completeness. However it affects less the CGS statistics because the latter includes CPO estimates with large bias but small uncertainty (see Fig. 39.3).
– Weighted ADEV estimate provides robust statistics for ranging CPO series which does not practically depend on low-frequency components of the time series. In particular, it can be mentioned that combined IVS series shows the least noise level.

One more result not included in these tables is that 2D WMADEV estimates are very close to the average of WADEV estimates computed for dX and dY multiplied by $\sqrt{2}$. Thus the multidimensional ADEV provides more compact expression for noise characteristics.

3.4 Comparison of CRF Realization

Noise level comparison of CPO time series computed with different radio source catalogues is one of very few absolute methods (maybe the only method proposed so far) for quality assessment of the CRF realizations. In Malkin (2008b), Sokolova and Malkin (2007), this method was used for the first time to compare several catalogues. Here we use it for comparison of the first and the second ICRF realizations. For this purpose, we computed two CPO series using ICRF1 (ICRF-Ext.2, Fey et al. 2004) and ICRF2 (Ma et al. 2009) source positions. Then we computed their scatter level using modified ADEV estimates WADEV and 2D WMADEV.

From the results presented in Table 39.4, one can see that the scatter of the CPO series computed with ICRF2 is substantially smaller than for series computed with ICRF1. We consider this result as clear evidence of better accuracy of the ICRF2 source positions as compared with the ICRF1.

4 Summary

The modified Allan variation (AVAR) and associated Allan deviation (ADEV) estimators proposed in Malkin (2008a) for processing unevenly weighted and/or multidimensional data is an effective and convenient tool for geodetic and astronomical time series scatter analysis. An important advantage of ADEV is its low sensitivity to low-frequency signal variations as compared to WRMS, which heavily depend on the model used to separate the stochastic and systematic signal components. However, both the original and the modified ADEV may inadequate estimate the noise level when jumps are present the time series.

The AVAR (ADEV) modifications, WAVAR (WADEV) and WMAVAR (WMADEV) developed to process unevenly weighted one- and multidimensional measurements, allow us to get noise characteristics that are less sensitive to outliers than the original ADEV estimate, provided the outliers have exaggerated uncertainties, which is usually the case. It is clear that the WMADEV (WMAVAR) definition is the most general as it includes the original AVAR definition and its modifications, WADEV and MADEV, as special cases.

For thoroughness' sake, it should be mentioned that the original AVAR is computed on evenly spaced time series. Unfortunately, it is not the case for many geodetic and astronomical applications. Many time series have regular time span however, e. g. daily or weekly station coordinates, daily troposphere parameters, etc. Other series can be made (near)-regular by averaging measured data over equal intervals. For example, such a method was used by Feissel-Vernier (2003) for radio source position time series. On the other hand, if AVAR and its modifications are used as a series scatter index, uneven spacing should not influence result of analsys, to our mind. And otherwise, averaging the original series may lead to loss of information.

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