ISSN 1063-7737, Astronomy Letters, 2014, Vol. 40, No. 5, pp. 268–277. © Pleiades Publishing, Inc., 2014.
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Pulkovo Combined Catalogue of Radio Source Positions PUL 2013

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Received May 30, 2013

Abstract—According to the decision of the International Astronomical Union (IAU), since 1998 the International Celestial Reference System has been realized by the ICRF catalogue of extragalactic radio source positions obtained from VLBI observations. Over the past years, the accuracy of the ICRF catalogue data has been increased only through an increase in the number and quality of observations and an improvement in the methods of their processing. Both the first ICRF version and the new ICRF2 version adopted by the IAU in 2009 are based on the catalogues obtained at the same VLBI data processing center. However, the experience of classical astrometry shows that a significant increase in the accuracy of the International Celestial Reference Frame can be achieved by creating combined catalogues, such as the fundamental catalogues of star positions. The same approach was applied to improve the ICRF catalogue. Even the first experience of such a combined solution has shown its high efficiency. Here, a new combined catalogue of radio source positions PUL(2013)C02 is presented. Mainly classical methods based on the expansion of the systematic differences between the input catalogues into series of orthogonal functions with additional improvements have been applied for its creation. Comparison of the combined catalogue obtained with the ICRF2 catalogue has shown that the latter is most likely not devoid of systematic errors at a level of 15–20 μas.

DOI: 10.1134/S1063773714050041

Keywords: *astrometry, reference frames, ICRF, VLBI.*

INTRODUCTION

The establishment of a celestial reference system and the realization of the corresponding reference frame in the form of a high-accuracy catalogue of positions and, in the necessary cases, proper motions of reference celestial objects is a traditional and main task of astrometry. According to the decision of the International Astronomical Union (IAU), since 1998 the International Celestial Reference System (ICRS) has been realized by the ICRF (International Celestial Reference Frame) catalogue of extragalactic radio source positions obtained by processing 1.6 millions of observations by the method of Very Long Baseline Interferometry (VLBI), which replaced the optical fundamental FK5 catalogue (Ma et al. 1998; Feissel and Mignard 1998). In other words, the ICRF is a celestial reference frame that practically realizes the ICRS. Although the ICRF catalogue contained 608 sources, its system was specified by the positions of 212 defining sources. The median position error in the ICRF catalogue was 0.65 mas for all sources and 0.45 mas for the defining sources.

In the time elapsed since then, several times more observations have been accumulated than was used for the ICRF creation and their processing algorithms have been improved significantly. As a result, in 2009 the second version of the radio-astrometric celestial reference frame, ICRF2, was created and adopted by the IAU General Assembly (Ma et al. 2009). Already 6.5 million observations in 1979–2009 were used for the ICRF2 catalogue. As a result, the ICRF2 catalogue contains 3414 sources, including 295 defining ones, with a median position error of 0.63 mas for all sources and 0.07 mas for the defining ones. It should be noted that the increase in the mean position error for all ICRF2 sources compared to the ICRF occurred due to the inclusion of 2197 sources from the VCS (VLBA Calibrator Survey) program observed in one or two sessions and, for this reason, having a considerably lower accuracy than the sources of regular radio-astrometric programs (Ma et al. 2009). Without these sources, the median error in the ICRF2 positions is 0.19 mas. However, an even more important advantage of the ICRF2 relative to the ICRF is a considerable decrease in its systematic errors, which will be discussed below. The ICRF2 catalogue, just

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Table 1. The radio source position catalogues used

Catalogue	Software	Period of observations, years	Number of sources
aus2012b	Occam	1980–2012	2892
bkg2011a	Calc/Solve	1984–2011	3214
cgs2012a	Calc/Solve	1980–2011	842
gsf2012a	Calc/Solve	1979–2012	3708
igg2012b	VieVs	1984–2012	860
opa2012a	Calc/Solve	1979–2012	3482
sha2012a	Calc/Solve	1979–2012	3470
usn2012a	Calc/Solve	1979–2012	793

as the ICRF one, was obtained at the NASA Goddard Space Flight Center.

The ICRF errors were investigated at the Pulkovo Observatory in 2006–2007. Comparison with the current catalogues of source positions regularly computed at several VLBI analysis centers (which we will call the individual or input ones) showed that by that time the ICRF catalogue had accumulated large systematic errors with an amplitude of 0.2–0.3 mas and with a complex structure (Sokolova and Malkin 2007). To rectify this situation, we constructed the combined catalogue RSC(PUL)07C02 based on individual catalogues, which is an improvement of the ICRF and, most importantly, in the systematic respect. We used methods similar to the methods of optical astrometry applied in compiling the fundamental catalogues (Sokolova and Malkin 2007).

In this paper, we present a new combined Pulkovo catalogue of radio source positions, Pul(2013)C02, which was constructed based on the latest individual catalogues, mainly according to the scheme applied by Sokolova and Malkin (2007) using improved methods for constructing a combined catalogue.

DESCRIPTION AND COMPARISON OF THE INPUT CATALOGUES

In this paper, we used eight individual catalogues obtained at VLBI analysis centers: AUS (Geoscience, Australia), BKG (Bundesamt für Kartographie und Geodäsie, Germany), CGS (Space Geodesy Center, Italy), GSF (Goddard Space Flight Center, NASA, USA), OPA (Paris Observatory, France), IGG (Vienna Technical University, Austria), SHA (Shanghai Observatory, People's Republic of China), and USN (US Naval Observatory). In the latter catalogue, we used only the source positions obtained in a global solution, i.e., the sources with their positions estimated from individual sessions were not included in the processing.

A list of the catalogues used is given in Table 1. The first three letters in the catalogue name denote the analysis center; the year in which the catalogue was obtained and the catalogue number in this year are specified next. The AUS, BKG, CGS, OPA, and GSF catalogues were taken from the IVS databases or the Internet resources of the analysis centers; the IGG and USN catalogues were kindly provided by their authors. Table 1 also gives the software used to compute the catalogue, the period of observations used, and the number of sources (with the VCS sources excluded). We used the ICRF2 as the main comparison catalogue at the main stages of our work.

The systematic differences between catalogues are usually investigated using some set of reference sources whose choice to some extent affects the final result. In this paper, for comparison, we used a set of 274 defining ICRF2 sources common to all individual catalogues. The distribution of these sources over the celestial sphere is shown in Fig. 1.

Figure 2 presents the weighted root-mean-square (WRMS) differences calculated from the formula

$$wrms = \left(\frac{\varepsilon^T Q_w^{-1} \varepsilon}{e^T Q_w^{-1} e}\right)^{1/2},$$

where e is a vector composed of ones, Q_w is the covariance matrix, and ε is the radio source coordinate difference vector (Titov 2001). The covariance matrix Q_w can be diagonal if the authors of the catalogue provide only the source position error, two-diagonal if the correlations between the right ascension and declination of each source are also given (the most commonly used format for the IVS centers), or full if given by the authors of the catalogue. The structure of this matrix is as follows:

$$\begin{pmatrix} \sigma^2_{\alpha_1} & \mathrm{cov}(\delta_1,\alpha_1) & \mathrm{cov}(\alpha_2,\alpha_1) & \mathrm{cov}(\delta_2,\alpha_1) & \ldots \\ & \sigma^2_{\delta_1} & \mathrm{cov}(\alpha_2,\delta_1) & \mathrm{cov}(\delta_2,\delta_1) & \ldots \\ & & \sigma^2_{\alpha_2} & \mathrm{cov}(\delta_2,\alpha_2) & \ldots \\ & & & \sigma^2_{\delta_2} & \ldots \\ & & & & \ldots \end{pmatrix},$$

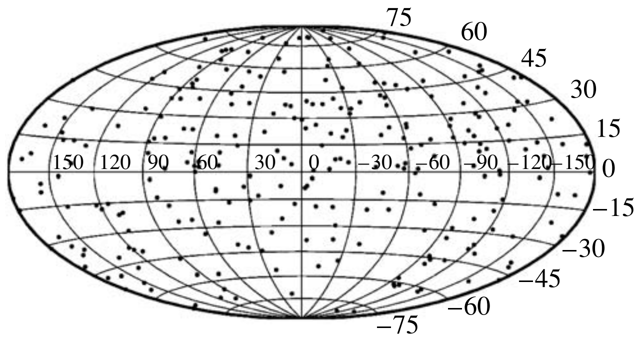

Fig. 1. Distribution of common reference sources over the celestial sphere (274 sources).

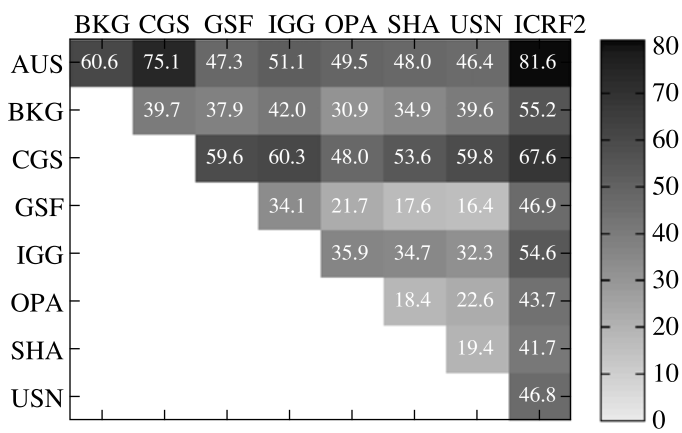

Fig. 2. WRMS differences between the input catalogues from 274 reference sources (jointly in $\Delta\alpha\cos\delta$ and $\Delta\delta$ μas).

CHOOSING AN OPTIMAL METHOD FOR REPRESENTING THE SYSTEMATIC DIFFERENCES

Apart from the official ICRF version (at present, ICRF2), the individual radio source position catalogues computed at various VLBI analysis centers are also accessible for usage. Although a constraint on the tie to the ICRF (in the sense of orientation) is applied in computing these catalogues, each such catalogue, in general, is its own celestial reference frame. Therefore, the question of comparing these catalogues arises. The main interest in the problem of comparing and subsequently combining the catalogues is to determine the systematic coordinate differences between these systems.

In this paper, we analyzed three models of the representation of systematic differences: the rigid rotation model, the spherical harmonic expansion (Brosche 1966), and the combined model of rigid rotation and Brosche's method, corresponding to the method of Schwan (2001).

The Rigid Rotation Model

Let there be two coordinate systems X_1, Y_1, $Z_1(\alpha_1, \delta_1)$ and X_2, Y_2, $Z_2(\alpha_2, \delta_2)$. The coordinate system α_2, δ_2 is formed from the coordinate system α_1, δ_1 by rotation around the X_1, Y_1, Z_1 axes through angles A_1, A_2, A_3. If the angles are small, then the following coordinate transformation is valid with an adequate accuracy:

$$\begin{pmatrix} \cos\alpha_2\cos\delta_2 \\ \sin\alpha_2\cos\delta_2 \\ \sin\delta_2 \end{pmatrix} = \begin{pmatrix} 1 & A_3 & -A_2 \\ -A_3 & 1 & A_1 \\ A_2 & -A_1 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha_1\cos\delta_1 \\ \sin\alpha_1\cos\delta_1 \\ \sin\delta_1 \end{pmatrix}.$$

Table 2. Parameters of the orientation between the individual catalogues and ICRS2 obtained using model (1)

Catalogue	A_1	A_2	A_3	WRMS_1	WRMS_2
aus2012b	-23.42 ± 6.28	6.75 ± 6.43	2.03 ± 5.68	81.6	80.5
bkg2011a	25.87 ± 4.12	17.6 ± 4.19	-10.73 ± 3.68	55.2	52.2
cgs2012a	13.49 ± 5.21	-4.30 ± 5.31	-18.04 ± 4.68	67.6	66.3
gsf2012a	-1.64 ± 3.65	8.99 ± 3.73	-2.14 ± 3.30	46.9	46.6
Igg2012c	13.45 ± 4.32	12.78 ± 4.43	-1.06 ± 3.65	54.6	537
opa2012a	-4.08 ± 3.36	12.46 ± 3.43	-9.43 ± 3.02	43.7	42.8
sha2012b	-2.89 ± 3.25	4.55 ± 3.32	-4.48 ± 2.92	41.7	41.5
usn2012a	-3.18 ± 3.62	12.38 ± 3.70	-4.19 ± 3.27	46.8	46.2

WRMS_1 are the weighted root-mean squares of the original differences, WRMS_2 are the weighted root-mean squares of the residuals once the systematic differences calculated with model (1), in microarcseconds, have been taken into account.

Denoting

$$\Delta\alpha = \alpha_1 - \alpha_2,$$
$$\Delta\delta = \delta_1 - \delta_2,$$

we obtain an expression for the systematic differences between two catalogues caused by rotation:

$$\Delta\alpha\cos\delta = A_1\cos\alpha\sin\delta + A_2\sin\alpha\sin\delta - A_3\cos\delta, \qquad (1)$$
$$\Delta\delta = -A_1\sin\alpha + A_2\cos\alpha.$$

The set of such equations (1) for all or selected common sources in two catalogues is solved by the least-squares method (LSM) to determine the mutual orientation angles A_1, A_2, and A_3 for the two reference frames specified by the catalogues being compared.

A significant shortcoming of model (1) is that it is implied in the solution that the quantities $\Delta\alpha$ and $\Delta\delta$ consist only of the rotational components and noise, while the systematic differences usually also contain the nonrotational components that can introduce distortions in determining the rotation parameters.

It should be mentioned that as the modern radio source position catalogues are computed, they are reduced to ICRF2 by imposing a constraint on the absence of rotation between the catalogues determined from 295 ICRF2 defining sources. Nevertheless, it can be seen from Table 2 that the orientation angles relative to ICRF2 turn out to be statistically significant for some of the catalogues, suggesting that, in general, each catalogue represents its own system and requires a further reduction to ICRF2. Why this occurs is the subject of a special study. One possible explanation is the influence of the covariance information on the orientation parameters. As was shown by Jacobs et al. (2010) and Sokolova and Malkin (2013), it is important to take into account the correlations between the source coordinates when determining the mutual orientation. The results obtained showed that the difference in the rotation angles determined with and without allowance for the correlation information can exceed 20 μas. Thus, the complete correlation information should be taken into account when comparing and combining the catalogues. Of course, this is possible only if all catalogues will be published with the full covariance matrix, which does not occur at present. Usually, only the correlations between the right ascension and declination are provided for each source. In this paper, we used all of the available correlation information.

Expansion in Orthogonal Functions

For a more accurate description of the systematic differences, their representation by orthogonal functions can be used. As in the previous cases, the individual differences in the positions of common

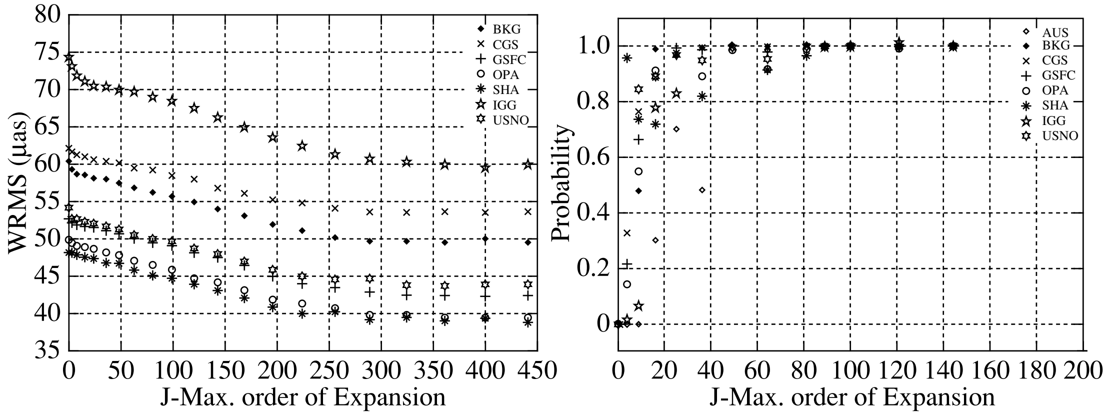

Fig. 3. WRMS versus the number of expansion terms (a) and Fisher's test (b). The closer the probability to unity, the less statistically justified is the increase in the number of expansion terms.

sources in two catalogues are the input data:

$$\begin{Bmatrix} \Delta\alpha_i \cos\delta \\ \Delta\delta_i \end{Bmatrix} = f\left(\alpha_i, \delta_i\right), \quad i = 1, 2..., N.$$

The generalized representation of these differences is

$$f\left(\alpha, \delta\right) = \sum_{j=0}^{g} b_j Y_j\left(\alpha, \delta\right) + \varepsilon, \qquad (2)$$

where Y_j is the complete set of orthogonal functions in the space formed by the variables α and δ, b_j are the expansion coefficients, and ε is the stochastic error.

Brosche's Model

In the method of Brosche (1966), the basis functions Y_j are

$$Y_j = K_j\left(\alpha, \delta\right),$$

where the spherical harmonics $K_j\left(\alpha, \delta\right)$ are specified by the expression

$$K_j\left(\alpha, \delta\right) = \begin{cases} P_{n0}\left(\delta\right), & k = 0, \\ P_{nk}\left(\delta\right)\sin\left(k\alpha\right), & k \neq 0,\ l = 0, \\ P_{nk}\left(\delta\right)\cos\left(k\alpha\right), & k \neq 0,\ l = 1, \end{cases}$$

where $P_{nk}\left(\delta\right)$ are the associated Legendre polynomials calculated from the formula

$$P_{nk}\left(\delta\right) = \cos^k\left(\delta\right)\left[\sin^p\left(\delta\right) + \sum_{\mu=1}^{[p/2]} \frac{(-1)^\mu \prod\limits_{\nu=0}^{2\mu-1}\left(p-\nu\right)}{\prod\limits_{\nu=1}^{\mu} 2\nu\left(2n-2\nu+1\right)} \sin^{p-2\mu}\left(\delta\right)\right],$$

$$n = 0, 1, 2, ...,$$
$$k = 0, 1, 2, ..., n,$$

where $p = n - k$, $[p/2]$ is the integer part of $p/2$ and the index j is related to the indices n, k, and l by the relation $j = n^2 + 2k + l - 1$.

Since the 274 sources used to determinate the systematic differences are distributed fairly uniformly

Table 3. Analytical representations of the expansion terms K_{nkl}

Indices		$Y_j = K_{nkl}$
n, k, l	j	
0, 0, 1	0	1
1, 0, 1	1	$\sin\delta$
1, 1, 0	2	$\cos\delta\sin\alpha$
1, 1, 1	3	$\cos\delta\cos\alpha$
2, 0, 1	4	$\sin^2\delta - 1/3$
2, 1, 0	5	$\cos\delta\sin\delta\sin\alpha$
2, 1, 1	6	$\cos\delta\sin\delta\cos\alpha$
2, 2, 0	7	$\cos^2\delta\sin 2\alpha$
2, 2, 1	8	$\cos^2\delta\cos 2\alpha$
...	...	...

Table 4. WRMS coordinate differences between the catalogues before and after the approximation of the systematic differences based on three different models

Model	AUS	BKG	CGS	GSF	IGG	OPA	SHA	USN
Input	81.6	55.2	67.6	46.9	54.6	43.7	41.7	46.8
Rotation	80.5	52.2	66.3	46.6	53.7	42.8	41.5	46.2
Brosche	73.9	47.4	49.1	42.9	46.7	38.7	37.8	41.6
Rotation + Brosche	74.1	47.0	53.8	42.7	46.9	38.8	37.9	41.3

Rotation is the rotation model (4), Brosche is Brosche's spherical harmonic expansion model, and the combined rotation + Brosche model (μas).

Table 5. WRMS differences between the input catalogues, ICRF2, and the combined catalogue for 274 common reference sources (μas)

Catalogue	AUS	BKG	CGS	GSF	IGG	OPA	SHA	USN	ICRF2
ICRF2	81.6	55.2	67.6	46.9	54.6	43.7	41.7	46.8	
PUL2013_C02	38.8	27.9	48.5	16.6	26.5	15.5	15.4	17.9	40.3

over the celestial sphere (see Fig. 1), we did not perform additional orthogonalization by the Gram–Schmidt method. One important problem of separating the stochastic and systematic parts of the coordinate differences is to estimate the maximum number of expansion terms. In this paper, we used Fisher's statistical test and a visual representation of the dependence of WRMSs on expansion term (see Fig. 3) to choose the maximum expansion term of Brosche's model.

Fisher's statistical test is based on comparing the sample variances of two or more variational series. The test was performed by comparing the value of the statistics with the critical value of the corresponding Fisher distribution at a given significance level. If the calculated empirical value of Fisher's test is greater than the critical value for a specific significance level and the corresponding numbers of degrees of freedom, then the variances are considered different. In other words, we test the hypothesis that the population variances of the populations considered are equal. In this paper, we applied the method of hypothesis testing using $p(F)$, the probability that the random variable with a given Fisher distribution exceeds a given critical value of the statistics. If $p(F)$ is smaller than the specified significance level α, then the null hypothesis is rejected and it is concluded that the expansion in $n + 1$ terms gives a statistically significant improvement compared to the expansion in n terms.

In general, comparison of each pair of catalogues yields its own optimal value of n. Below, however, the derived expansion coefficients are used in obtaining the combined catalogue (Sokolova and Malkin 2007). In this case, the number of expansion terms should be the same for all catalogues. Therefore, after a preliminary analysis of the differences of each input catalogue with ICRF2 and obtaining the optimal value of n for each of them for the final computations, we used the maximum of these values. Thus, we took 36 expansion terms for Brosche's model.

As was shown in previous studies (Vityazev 1997, 1999; Schwan 2001), using only the spherical harmonic expansion, in general, is insufficient for a complete representation of the systematic differences, because, as can be seen from Table 3, it does not contain explicitly the terms responsible for the rotation. Therefore, we added a version of the joint use of coefficients (1) and (2), corresponding to the model of Schwan (2001).

Table 4 lists the WRMS differences between all of the individual catalogues used in this paper and ICRF2 (first row) and the residuals after the elimination of the systematic errors determined by the three methods. As can be seen from these data, in this case,

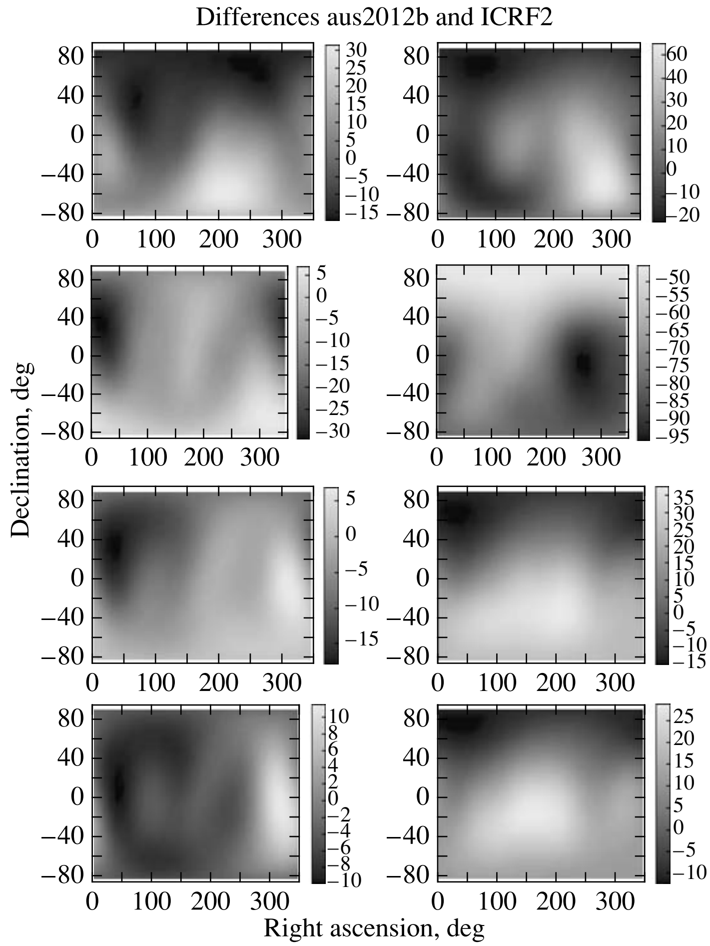

Fig. 4. Smoothed differences between the individual catalogues and ICRF2 in mas from 274 reference sources ($\Delta\alpha \cos\delta$ on the left, $\Delta\delta$ on the right).

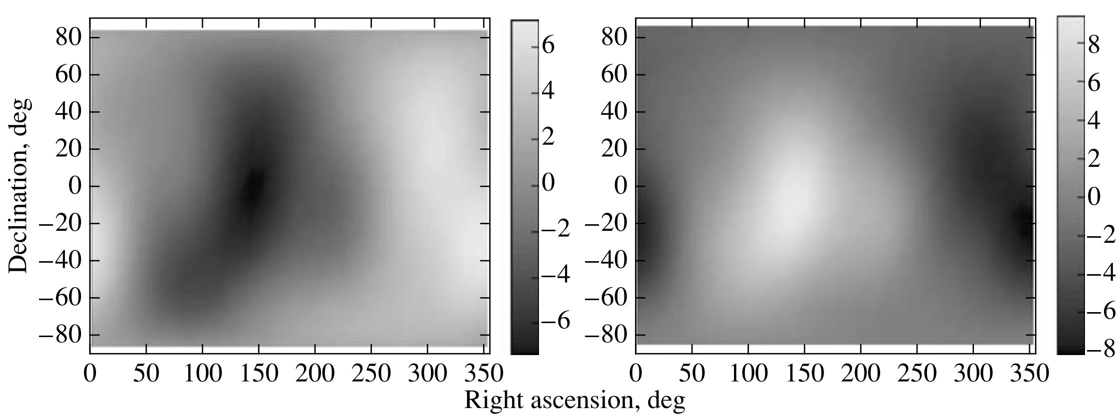

Fig. 5. Smoothed differences between the Pul(2013)C01 and ICRF2 catalogues in μas from 274 reference sources ($\Delta\alpha \cos\delta$ on the left, $\Delta\delta$ on the right).

the representation of the systematic differences by the expansion in spherical harmonics gives WRMSs that are, on the whole, similar to those in Schwan's model (Rotation + Brosche) due to the smallness of the rotation angles (A_1, A_2, A_3).

It can also be seen from our analysis that the catalogues obtained using the Calc/Solve package have similar and the smallest deviations from ICRF2. This is most likely explained by the fact that the orig-

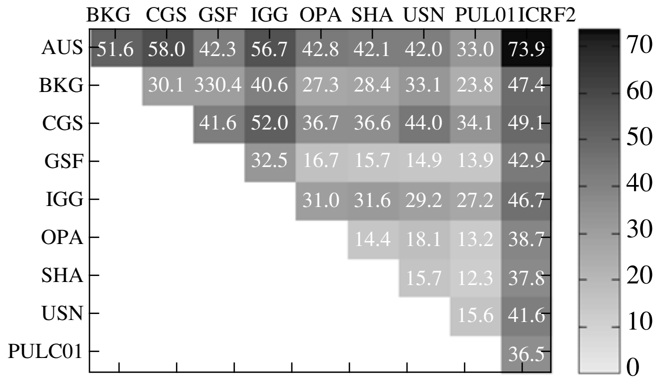

Fig. 6. WRMS coordinate differences between the catalogues reduced to ICRF2 and the Pul(2013)C01 combined catalogue in μas obtained from 274 reference sources (jointly in $\Delta\alpha\cos\delta$ and $\Delta\delta$).

inal version of the ICRF2 catalogue was constructed using this package, which is the most developed one among the present-day VLBI processing packages. The deviations of the AUS and IGG catalogues from ICRF2 can be due to the differences in processing packages and estimation methods. The large values of the WRMS differences with ICRF2, which suggest systematic differences of all the input catalogues with the official IAU system, arouse the greatest interest. Figure 4 presents the smoothed differences between some of the individual catalogues and ICRF2 constructed from 274 reference sources.

STOCHASTIC IMPROVEMENT OF THE ICRF2 CATALOGUE

The next step is a stochastic improvement of the ICRF2 catalogue. For this purpose, all of the input catalogues were reduced to ICRF2 using Brosche's method and, for comparison, the combined "Rotation + Brosche" model. Note that the systematic differences between each input catalogue and ICRF2 were computed from 274 common sources, as has been pointed out above, and the system found was used to reduce all sources of the input catalogue to ICRF2. Note also that to subsequently find the average system, we used the same number of expansion terms for all individual catalogues.

We then computed the weighted mean system of Pul(2013)C01 source coordinates (Brosche's method was used) in ICRF2. This catalogue, which is a stochastic improvement of the ICRF, includes a total of 2793 sources. The condition for including a source in the combined catalogue was its presence at least in two individual catalogues. The Pul(2013)C01 catalogue is compared with ICRF2 in Fig. 5, from which it can be seen that the catalogue reproduces ICRF2 with an accuracy better than 10 μas. Figure 6 presents the WRMS differences between the input catalogues reduced to ICRF2 and the combined catalogue.

SYSTEMATIC IMPROVEMENT OF THE ICRF2 CATALOGUE

The final step in constructing the combined catalogue is a systematic improvement of ICRF2. For this purpose, we computed the average system from the approximating functions for the input catalogues by the chosen method (Brosche), at the first step without using any weights:

$$\begin{Bmatrix} \Delta\alpha_i' \\ \Delta\delta_i' \end{Bmatrix} = \frac{1}{M}\sum_m \sum_j b_j^{im} K_j^{im}(\alpha_i^m, \delta_i^m).$$

We compared the system constructed in this way with the individual systems on a $10°(\alpha) \times 5°(\delta)$ grid and determined the root-mean-square deviation (RMS) of each individual system from the mean. The RMS values obtained determined the weight of each catalogue at the next iteration. The derived weighted mean system was added to the combined Pul(2013)C01 catalogue in ICRF2; as a result, the Pul(2013)C02 catalogue was obtained. Figure 7 presents the smoothed differences between the Pul(2013)C02 and ICRF2 catalogues.

Table 5 gives the WRMS difference between the input catalogues, ICRF2, and the combined catalogues for 274 common reference sources. Comparison shows that the ICRF2 catalogue (Ma et al. 2009) may be not devoid of significant systematic errors at a level of 15–20 μas.

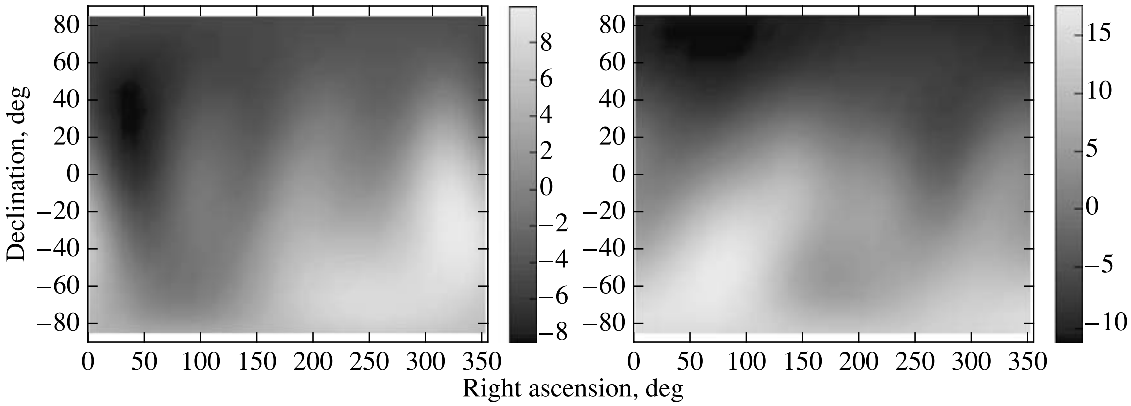

Fig. 7. Smoothed differences between Pul(2013)C02 and ICRF2 in μas obtained from 274 reference sources ($\Delta\alpha\cos\delta$ on the left, $\Delta\delta$ on the right).

CONCLUSIONS

In this paper, we presented a new combined catalogue of positions for 2793 radio sources, Pul(2013)C02. For its computation, we investigated and used methods for the construction of combined catalogues similar to those that were applied in optical astrometry to construct the fundamental star catalogues. We used eight catalogues obtained at different IVS analysis centers as the input ones.

To choose the optimal technique for constructing a combined catalogue, we compared a number of analytical methods for representing the systematic differences between the catalogues. As a result of this comparison, we showed that the methods of expansion in orthogonal functions for most of the individual catalogues allow the systematic differences to be represented better than does the rigid rotation method. Due to the smallness of the derived rotation angles, we chose Brosche's method as the best method for representing the systematic differences.

As an intermediate one, we obtained the Pul(2013)C01 combined catalogue computed as a weighted mean of the input catalogues reduced to ICRF2. This catalogue is of interest in its own right, because it is a stochastic improvement of ICRF2.

The final Pul(2013)C02 catalogue was obtained from the Pul(2013)C01 catalogue by taking into account the average system of input catalogues. Thus, this catalogue is both stochastic and systematic improvement of the ICRF2 catalogue.

Comparison of the combined catalogue with ICRF2 shows that ICRF2 can have significant systematic errors at a level of 15–20 μas. The most probable cause of these errors can be the insufficiently reliable observations of several radio sources at the time of ICRF2 compilation. In three years elapsed since that time, about 1.5 million new observations have been obtained, which have allowed the positions of many sources to be improved. This conclusion is also confirmed by the existence of significant systematic differences between the individual catalogues and ICRF2 (Fig. 4). However, these differences are significantly different in form for different catalogues. The combined catalogue allows an average, most justified estimate of the systematic errors of ICRF2 to be obtained. Hence it follows that to maintain the systematic accuracy of the ICRF at a level of 10–15 μas requires updating it with an interval of no more than five years, as was proposed by Malkin (2013).

At present, there are various views of the methods for constructing and improving the ICRF catalogue. In this connection, a further development of the objective methods for estimating the quality of individual radio source position catalogues and improving the methods for constructing the next ICRF versions seems important. As follows from our studies and the experience of constructing the celestial reference frames based on optical observations of stars, the construction of a combined catalogue is an optimal method for solving this problem. The experience of our first combined catalogue (Sokolova and Malkin 2007) serves as an additional basis for this conclusion. The systematic errors of the ICRF catalogue that we then obtained were subsequently confirmed when creating ICRF2 and comparing it with the ICRF.

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Translated by G. Rudnitskii