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On the Accuracy of the Theory of Precession and Nutation

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Received August 26, 2013; in final form, December 23, 2013

Abstract—Corrections to the IAU 2000/2006 parameters of the theory of precession and nutation are calculated using five different series—two individual series and three combined series that have been used in the literature for this purpose. A comparison of the sets of corrections obtained using the different datasets indicates significance systematic differences between them, which often substantially exceed the corresponding random errors. At the same time, existing studies have usually used data obtained from one or two series chosen by the authors without special justification. When refining the theory of precession and nutation, it is necessary to consider and compare various available series of VLBI data if one wishes to reduce the systematic errors in an improved model.

DOI: 10.1134/S1063772914060043

1. INTRODUCTION

Modern theories of the Earth's rotation, in particular modern models for precession and nutation, are constructed in two stages. In the first, a theoretical model based on our available knowledge of the structure of the Earth, geophysical processes occurring in its envelopes, the motion of celestial bodies that influence the Earth's rotation, and so forth is constructed (see, e.g., [1–3]). However, this knowledge is insufficient to construct a theory of the Earth's rotation with the microarcsecond (μas) accuracy that is required for modern applications. Therefore, a number of the main parameters describing the Earth's precession and nutation must be refined using observations, as was done for the last theory of precession and nutation officially adopted by the International Astronomical Union, IAU 2000/2006 [4, 5].

Starting in the 1980s, series of coordinates of the celestial pole obtained using Very Long Baseline Interferometry (VLBI) methods were used. Essentially, the results of VLBI observations are used to calculate, not the coordinates of the celestial pole, but corrections to these coordinates assumed in the adopted precession and nutation model, called the Celestial Pole Offset (CPO) dX and dY [6].

The input CPO series have been calculated by the International VLBI Service for Geodesy and Astrometry (IVS) [7]. In addition, combined CPO series have been published by the IVS and the International Earth Rotation and Reference Systems Service (IERS) [8]. More than ten CPO series are available for studies aiming to refine the theory of precession and nutation. The comparative study carried out in [9] showed the existence of significant systematic and random differences between these series. These differences inevitably influence the accuracy of the parameters of precession and nutation models derived from observations, and bring about dependences of these parameters on the CPO series used. Our current study investigates these effects.

2. DATA USED

The CPO series were first calculated by the IVS directly via the processing of VLBI observations. They contain the CPO values dX and dY for each 24-hour VLBI session at the time corresponding of the middle of the session, with, on average, about three observing epochs per week. We will call such series "individual". Even if different data-analysis teams use the same data from the IVS network, the results can differ quite substantially due to differences in the software and analysis methods used [9].

In all, nine individual CPO series are available.[1] However, only six of these provide rigorously calculated corrections to the IAU 2000/2006 model. Of these, we chose two series that include CPO values relative to the IAU 2003/2006 model, which were used in the creation of the IAU 2000/2006 model [4, 5]: the GCFC and NEOS CPO series. Both of these series were calculated using the Calc/Solve software

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[1] ftp://cddis.gsfc.nasa.gov/vlbi/ivsproducts/eops/

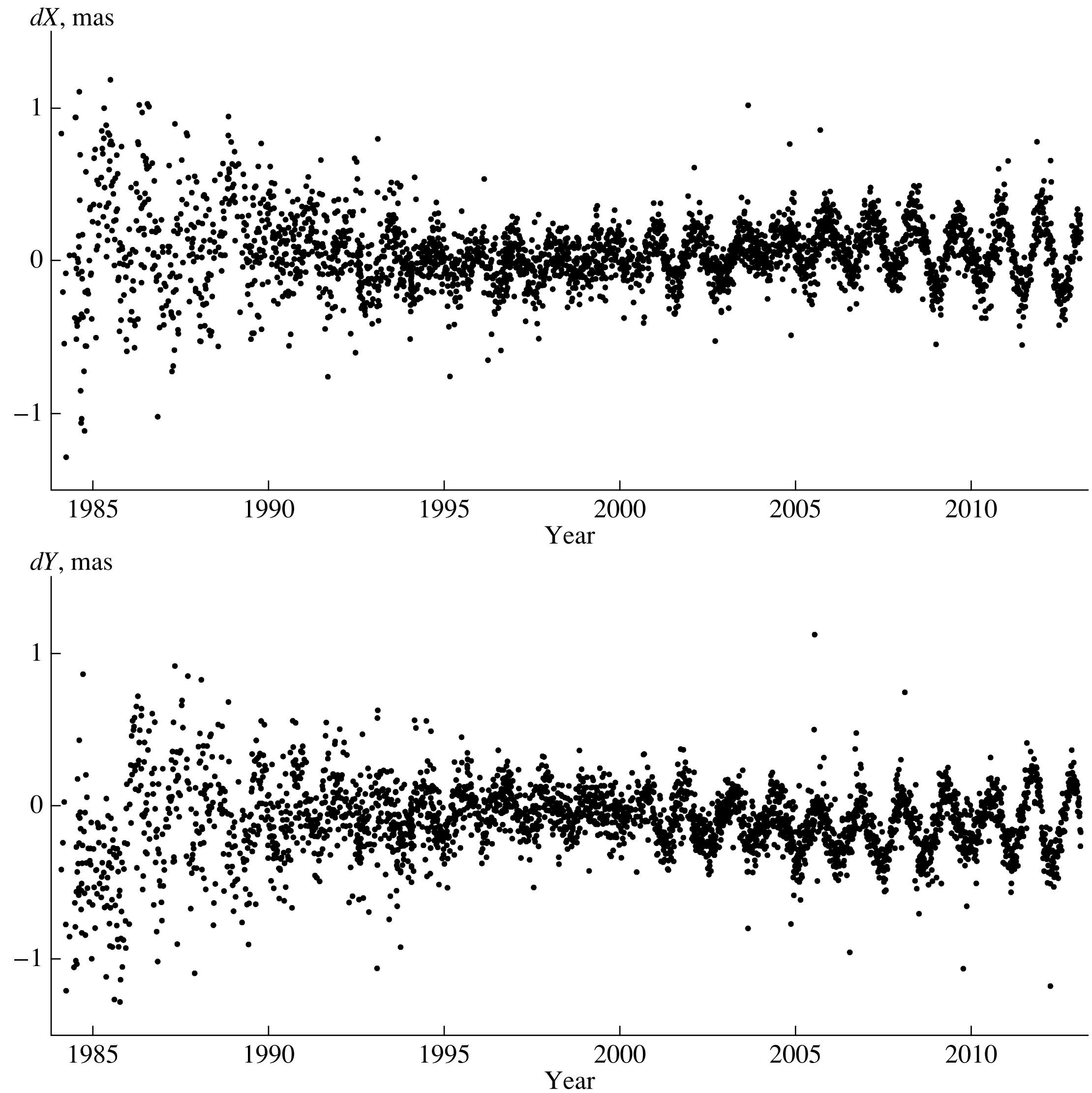

Fig. 1. The IVS CPO series.

package, at the NASA Goddard Space Flight Center and the US Naval Observatory, respectively.

Further, the individual series were used to obtain combined solutions at the analysis centers. The main combined CPO series are the IERS/C04 [10], IERS/NEOS [11], and IVS [12] series. Like the individual series, the IVS series contains CPO values for individual VLBI sessions at their central epochs. Fewer sessions were used in this series than in the individual series, on average, about 2.5 per week. The IERS series was calculated for each day at 0^h UT. Different sets of individual series and combination methods were used for the C04 and NEOS series.

The dX and dY components of the IVS CPO series are presented in Fig. 1. This figure shows that the main components of the CPO series correspond to quasi-harmonic oscillations with a period of about 430 days and with a variable amplitude, corresponding to free nutation of the Earth's core (FCN, Free Core Nutation), and a signal with a period of 18.6 yr due to errors in the model for the main nutation term.

The FCN signal has a variable amplitude and phase (see, e.g., [13]), hindering its inclusion in the precession and nutation model, and must be determined from observations. At the same time, it is the most substantial unmodeled component of the CPO, and appreciably exceeds the possible errors of the nutation model in amplitude. Therefore, the FCN signal must be excluded from the CPO series before computing corrections to the IAU 2000/2006 model.

At present, three FCN models have been developed and are kept up to date, and are accessible in numerical form: the models of Malkin ZM1 and ZM3 and the model of Lambert (see [6]). These models

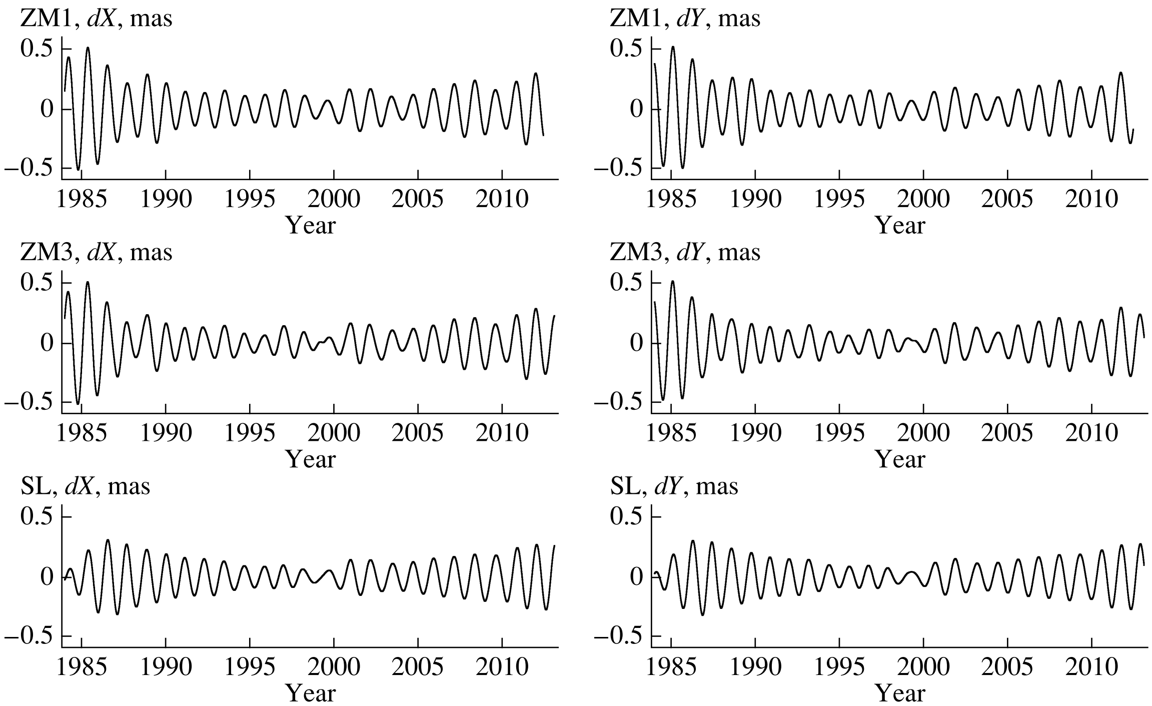

Fig. 2. The FCN series.

are described in detail in [13–15]. Comparisons of these models carried out in the above papers have shown the existence of differences between them, although they are not very large. These differences are due to the different input CPO series and different computation methods used to calculate the FCN models. The difference between the FCN models is large only in the 1980s, when the CPO measurements still had comparatively low accuracy in a random and, more importantly, systematic sense, as can be seen in Fig. 1. No substantial systematic differences between these models are observed for the period following 1990. All three FCN series are shown in Fig. 2. The ZM1 series is shorter than the other two series, since it was calculated using a wavelet transform possessing appreciable edge effects. Therefore, the last year and a half of the ZM1 series was eliminated, as it was considered unreliable.

We used the ZM3 FCN series[2] in our current study, which was calculated using a method that is close to that of Lambert. A comparison of the three models carried out in previous studies suggests that this model is preferable, since it is based on the most accurate IVS CPO series and provides the most detailed representation of the time behavior of the FCN [15]. The contribution of the FCN to the CPO series based on the ZM3 model was calculated as follows. We adopted the IVS CPO series as our input data. The FCN parameters were calculated using a sliding interval with a duration of about one FCN period (431 solar days), with a shift of one day. A system of conditional equations of the following form was composed in each interval:

$$\begin{aligned} dX &= A_c \cos\varphi - A_s \sin\varphi + X_0, \\ dY &= A_c \sin\varphi + A_s \cos\varphi + Y_0, \end{aligned} \tag{1}$$

where dX and dY are the observed CPO values (using the IVS series), $\varphi = \frac{2\pi}{P}(t - t_0)$, and P is the nominal FCN period, taken to be −430.21 solar days, with $t_0 = 2000.0$. One equation corresponds to a single VLBI session with a central observing epoch t. The weights of these conditional equations were taken to be inversely proportional to the squared CPO errors.

The parameters of the FCN model A_c, A_s, X_0, and Y_0 were calculated at the center of the current interval from this system of equations via a least-squares fit. When the sliding interval is shifted by a day, the parameter values are also obtained with a daily interval. The FCN values at an arbitrary epoch can be obtained from formulas analogous to (1), but without taking into account the shifts X_0 and Y_0, interpolating the values A_c and A_s to the required time (in practice, it is sufficient to take the parameter values at the beginning of the day containing the epoch at which the FCN contribution is calculated).

[2] `http://www.gao.spb.ru/english/as/persac/`

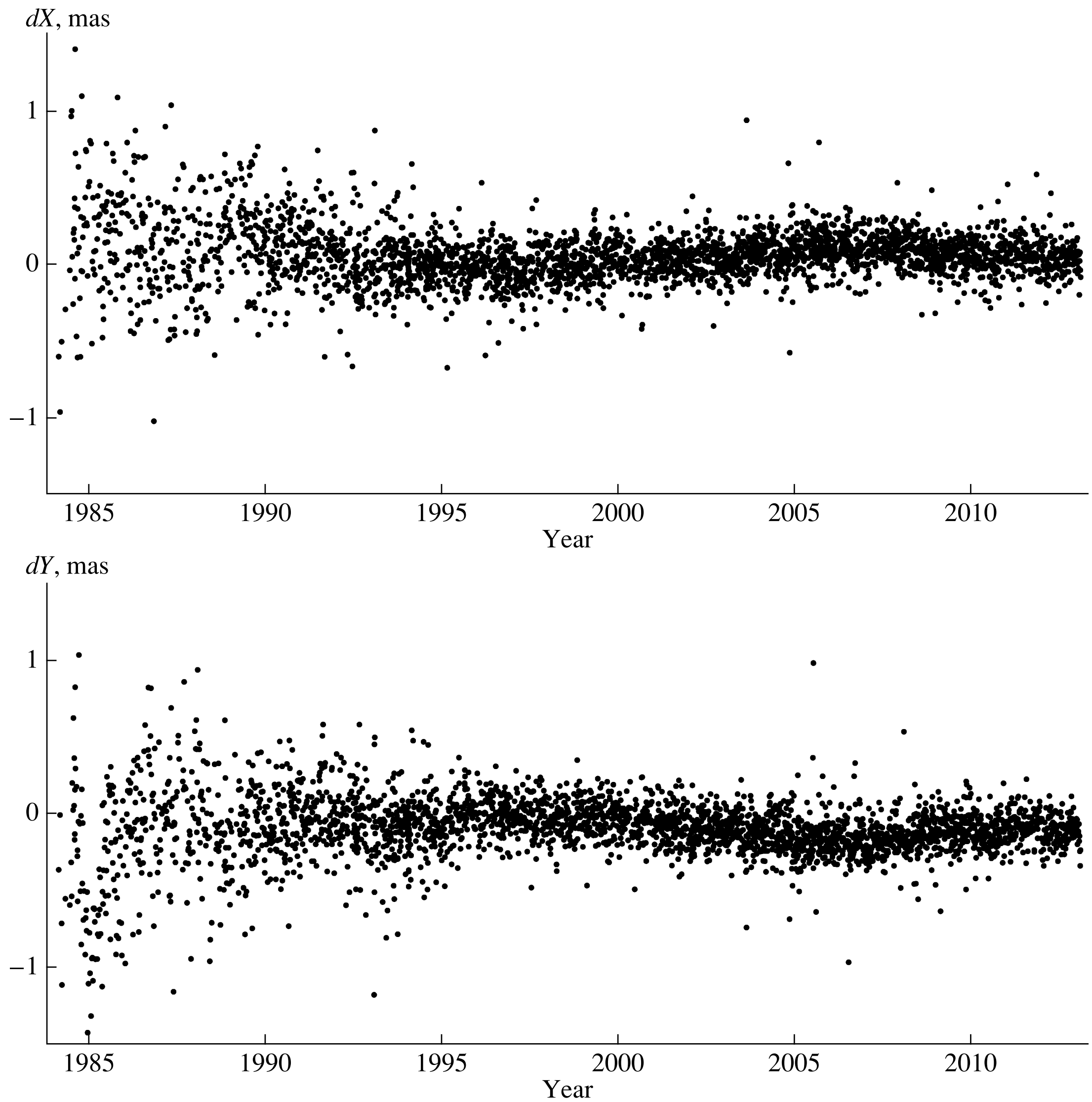

Fig. 3. IVS CPO series after excluding the FCN.

The IVS CPO series for the ZM3 model after excluding the FCN contribution (the CPO−FCN series) is shown in Fig. 3. This FCN model was also subtracted from the other CPO series reduced in our study. Plots of the CPO and CPO−FCN series from other data centers have a similar appearance, although detailed analyses show the existence of significant differences between them [14, 16]. Therefore, the use of different series in different studies aimed at refining the precession and nutation models could lead to differences in the results obtained, as will be investigated in the next section.

3. DETERMINING CORRECTIONS TO THE IAU 2000/2006 MODEL

We calculated corrections to the IAU 2000/2006 precession and nutation model using all five CPO−FCN series described in the previous section. The corrections to the precession parameters in the form of linear or quadratic terms and the amplitudes of the seven highest nutation harmonics were determined jointly. The CPO−FCN data were assigned weights that were inversely proportional to their squared errors. In addition to these five different CPO series, we also investigated the dependence of the derived corrections to the precession and nutation theory on the observational interval used.

As was noted above, the data obtained before approximately 1990 were comparatively sparse, displayed large scatter, and may have had large systematic errors, especially in dY. The availability of more homogeneous data begins from the early 1990s, as is reflected in the analysis of the random CPO errors shown in Fig. 4. All these features are also present

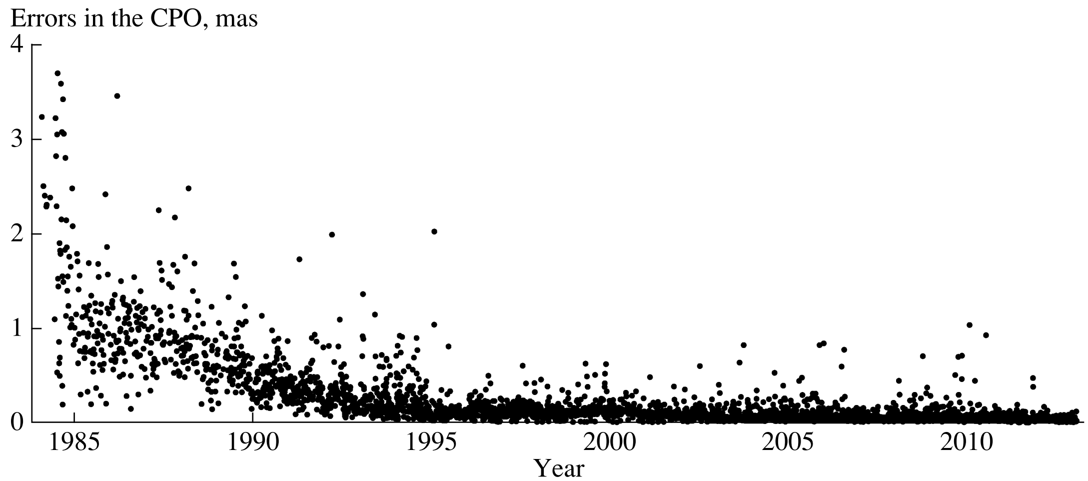

Fig. 4. Errors in the IVS CPO series.

in the other CPO series. Therefore, we can consider three possible dates for the beginning of the observing interval to be used to determine corrections to the precession and nutation model.

The first of these epochs, 1984.0, corresponds to the beginning of the IVS series. As was shown above, data taken before 1990 have comparatively low accuracy in both a random and systematic sense. Therefore, we do not consider their use to investigate fine effects manifest in the Earth's rotation to be expedient. However, they are often used to represent the beginning of the era of VLBI observations for various astronomical and geodetic studies. We therefore considered these data, to estimate the possible influence of the early observations during 1984–1990 on the results. The epoch 1990.0 has also recently begun to be used as the beginning of the period of more precise and uniform VLBI data. Differences in the precession parameters (for a linear precession model) when using VLBI data with these two beginning dates were already noted in [17]. The third possible beginning epoch—1993.0, which corresponds to the beginning of the period of better-quality, modern VLBI data—is being considered here for the first time. Figure 4 shows that the modern period of stable, high-precision CPO-determination results begins roughly from this epoch.

Thus, we carried out our computations using three data intervals determined by the three beginning dates indicated above. This makes it possible to estimate possible systematic discrepancies between the results of different studies that used VLBI-data intervals beginning from epochs 1984.0 and 1990.0. The end date for all three intervals was taken to be 2013.1.

Note that, if the data taken before the beginning of 1990.0 differed from the later data only in having larger random errors in the coordinates of the celestial pole, it would be useful to include these data to determine corrections to the precession constant and the long-period nutation terms, since this would increase the duration of the series, which is important for enhancing the accuracy with which the parameters of the trend and harmonics with periods comparable to the length of the series can be determined. If systematic errors are also present in the early observations, including these data could lead to systematic errors in the results. Applying weights to the CPO values in accordance with their errors cannot fully compensate for this effect. This leads to the need to find a compromise between the length of the series and the accuracy of the observational data used. A comparison of the results obtained for different observational intervals can help investigate the influence of the early observations and obtain a better impression of the real errors in the IAU precession and nutation model.

We determined corrections to the nutation model using the seven main terms, with periods of 6798.38, 3399.19, 365.26, 182.62, 121.75, 27.55, and 13.66 days—in all, 14 parameters (the amplitudes of the sine and cosine terms for each harmonic). The corrections to two versions of the precession model were calculated jointly with the corrections to the nutation model: a linear model, $p_0 + p_1(t - t_0)$, and a quadratic model, $p_0 + p_1(t - t_0) + p_2(t - t_0)^2$, where t is the time in years and $t_0 = 2000.0$. The quantity p_0 is the shift in the position of the celestial pole at epoch 2000.0, and p_1 is the correction to the presession rate. Thus, we obtained six solutions for each CPO–FCN series: solutions for linear and quadratic models for the corrections to the precession model (together with the nutation corrections, 16 and 17 parameters in all,

Table 1. Maximum correlation between the precession and nutation parameters (absolute values). The period of the corresponding nutation term is indicated for the sine and cosine coefficients. Each set of three rows in the table presents data for the initial epochs 1984.0, 1990.0, and 1993.0, from top to bottom.

	Linear precession model					Quadratic precession model					
	p_0	p_1	sin(6798)	cos(6798)	sin(3399)	p_0	p_1	p_2	sin(6798)	cos(6798)	sin(3399)
p_1	0.77					0.48					
	0.78					0.63					
	0.90					0.84					
p_2						0.80	0.90				
						0.87	0.94				
						0.93	0.97				
sin(6798)	0.30	0.36				0.71	0.84	0.91			
	0.25	0.29				0.81	0.90	0.95			
	0.32	0.37				0.88	0.96	0.98			
cos(6798)	0.52	0.75	0.20			0.66	0.38	0.70	0.51		
	0.41	0.75	0.24			0.74	0.51	0.80	0.66		
	0.65	0.85	0.31			0.85	0.72	0.83	0.76		
sin(3399)	0.31	0.49	0.48	0.40		0.49	0.24	0.42	0.37	0.54	
	0.31	0.49	0.46	0.40		0.55	0.39	0.58	0.36	0.58	
	0.38	0.53	0.39	0.44		0.70	0.62	0.69	0.60	0.71	
cos(3399)	0.24	0.26	0.14	0.36	0.10	0.49	0.44	0.52	0.47	0.58	0.14
	0.26	0.37	0.21	0.41	0.10	0.59	0.56	0.60	0.58	0.65	0.24
	0.38	0.43	0.13	0.48	0.19	0.73	0.75	0.80	0.80	0.60	0.34

respectively) for the three beginning epochs for the observational interval.

Due to the finite length of the observational interval, we expected a strong correlation between the coefficients of the precession model and the long-period nutation terms, as was already noted in earlier studies, such as [18]. We carried out a more detailed analysis of these correlations using the data considered here. Table 1 presents the maximum correlation coefficients for the correlations between the parameters of the precession model and the two longest-period nutation terms, derived using all five series. The correlations with the high-frequency nutation terms are weak, and are not presented in Table 1. As expected, the correlation between the derived parameters grows as the observational interval decreases, especially between the linear and quadratic precession coefficients and between the precession coefficients and the nutation coefficients for the term with a period of 6798 days. The corrections to the precession parameters for the quadratic precession model do not distinguish clearly between the linear and quadratic terms, but the correlations between the shift in the pole p_0 and the parameters of the nutation model are lower for the quadratic than for the linear precession model. Replacing the linear with the quadratic precession model appreciably changes the correlations not only between these parameters, but also between the nutation parameters.

The corrections to the precession terms obtained for the five CPO series are presented in Table 2. Results obtained for the three initial epochs T_1 are presented for each series. We used the combined estimate proposed in [19] to calculate the input values over all five series, which makes it possible to take into account both the errors in the input values and their scatter.

The results of our calculations show that both parameters describing the linear trend in the linear precession model are determined fairly certainly. Only the corrections to the pole shift p_0 are determined confidently in the quadratic model, while the corrections to the precession rate are less certain. Overall, the uncertainties in the parameters of the linear model

Table 2. Corrections to the precession parameters

Series	T_1	p_0 μas	p_1 μas/yr	p_0 μas	p_1 μas/yr	p_2 μas/yr^2
				dX		
GSF	1984	$+41.8 \pm 2.0$	$+1.05 \pm 0.31$	$+21.1 \pm 3.0$	-1.04 ± 0.39	$+0.62 \pm 0.07$
	1990	$+35.6 \pm 2.1$	$+2.42 \pm 0.34$	$+32.3 \pm 4.0$	$+1.83 \pm 0.70$	-0.12 ± 0.13
	1993	$+37.6 \pm 2.5$	$+1.93 \pm 0.47$	$+19.7 \pm 6.3$	-4.57 ± 2.12	$+0.96 \pm 0.31$
USN	1984	$+32.6 \pm 1.9$	$+1.92 \pm 0.35$	$+24.2 \pm 2.9$	$+1.31 \pm 0.35$	$+0.24 \pm 0.06$
	1990	$+30.2 \pm 2.0$	$+2.48 \pm 0.68$	$+27.5 \pm 4.1$	$+2.02 \pm 0.68$	$+0.10 \pm 0.13$
	1993	$+31.6 \pm 2.5$	$+2.12 \pm 2.17$	$+14.5 \pm 6.7$	-3.67 ± 2.17	$+0.89 \pm 0.32$
IVS	1984	$+27.1 \pm 2.6$	$+1.89 \pm 0.31$	$+14.1 \pm 4.0$	-2.15 ± 1.06	$+0.63 \pm 0.15$
	1990	$+26.3 \pm 2.8$	$+2.05 \pm 0.35$	$+12.3 \pm 4.9$	-2.90 ± 1.49	$+0.74 \pm 0.21$
	1993	$+26.9 \pm 3.1$	$+1.92 \pm 0.50$	-0.5 ± 6.1	11.24 ± 2.61	$+1.76 \pm 0.34$
C04	1984	$+36.2 \pm 2.4$	$+1.18 \pm 0.32$	$+30.8 \pm 3.9$	$+1.10 \pm 0.32$	$+0.10 \pm 0.06$
	1990	$+34.5 \pm 2.8$	$+1.50 \pm 0.40$	$+23.9 \pm 5.3$	$+0.40 \pm 0.61$	$+0.29 \pm 0.12$
	1993	$+28.8 \pm 4.7$	$+2.56 \pm 0.82$	$+21.4 \pm 6.8$	-2.30 ± 3.31	$+0.59 \pm 0.39$
NEOS	1984	-26.0 ± 1.5	$+3.49 \pm 0.18$	-19.6 ± 2.1	$+3.27 \pm 0.19$	-0.11 ± 0.03
	1990	-26.7 ± 1.1	$+3.62 \pm 0.16$	-39.1 ± 2.0	$+1.85 \pm 0.29$	$+0.43 \pm 0.06$
	1993	-25.6 ± 1.2	$+3.34 \pm 0.22$	-69.0 ± 3.1	10.67 ± 0.95	$+2.19 \pm 0.14$
Average		$+3.6 \pm 8.0$	$+2.73 \pm 0.26$	-5.5 ± 8.7	$+1.58 \pm 0.60$	$+0.15 \pm 0.11$
				dY		
GSF	1984	-89.0 ± 2.0	-1.42 ± 0.31	-94.3 ± 3.1	-1.95 ± 0.39	$+0.16 \pm 0.07$
	1990	-91.2 ± 2.1	-0.96 ± 0.36	-87.7 ± 4.1	-0.34 ± 0.72	-0.13 ± 0.13
	1993	-92.8 ± 2.6	-0.58 ± 0.49	-55.2 ± 6.3	$+13.17 \pm 2.17$	-2.03 ± 0.31
USN	1984	-81.2 ± 2.0	-1.46 ± 0.32	-75.4 ± 3.0	-1.07 ± 0.35	-0.16 ± 0.06
	1990	-80.0 ± 2.1	-1.74 ± 0.37	-80.8 ± 4.3	-1.88 ± 0.71	$+0.03 \pm 0.14$
	1993	-83.4 ± 2.6	-0.89 ± 0.52	-41.8 ± 6.9	$+13.31 \pm 2.24$	-2.17 ± 0.33
IVS	1984	-77.1 ± 2.6	-3.20 ± 0.47	-65.6 ± 4.0	$+0.31 \pm 1.02$	-0.55 ± 0.14
	1990	-76.9 ± 2.8	-3.26 ± 0.50	-60.3 ± 4.8	$+2.57 \pm 1.47$	-0.87 ± 0.21
	1993	-75.7 ± 3.2	-3.52 ± 0.58	-47.4 ± 6.1	$+10.52 \pm 2.64$	-1.86 ± 0.34
C04	1984	-86.0 ± 2.0	-0.98 ± 0.28	-78.8 ± 3.4	-0.93 ± 0.28	-0.13 ± 0.05
	1990	-84.2 ± 2.3	-1.25 ± 0.35	-101.9 ± 4.5	-3.10 ± 0.53	$+0.49 \pm 0.11$
	1993	-98.1 ± 3.9	$+1.44 \pm 0.69$	-90.3 ± 5.9	$+6.11 \pm 2.72$	-0.58 ± 0.33
NEOS	1984	-89.1 ± 1.9	-5.50 ± 0.19	-94.0 ± 2.8	-5.48 ± 0.19	$+0.07 \pm 0.03$
	1990	-96.6 ± 1.7	-5.15 ± 0.18	-118.0 ± 2.7	-7.59 ± 0.30	$+0.59 \pm 0.06$
	1993	-11.25 ± 1.9	-2.70 ± 0.25	-97.5 ± 3.4	$+2.40 \pm 1.09$	-0.73 ± 0.15
Average		-88.9 ± 2.6	-3.08 ± 0.52	-87.1 ± 5.1	-3.60 ± 0.81	$+0.04 \pm 0.09$

Table 3. Corrections to the amplitudes of the nutation terms determined jointly with the corrections to the linear precession model, in μas

Series	T_1	Harmonic period, days						
		6798.38	3399.19	365.26	182.62	121.75	27.55	13.66
					dX			
GSF	1984	63.7 ± 2.6	8.6 ± 2.3	3.7 ± 2.2	12.0 ± 2.2	1.6 ± 2.2	21.2 ± 2.2	24.3 ± 2.2
	1990	54.8 ± 2.8	9.7 ± 2.4	2.2 ± 2.2	11.8 ± 2.3	1.9 ± 2.2	21.4 ± 2.2	24.3 ± 2.2
	1993	57.3 ± 3.3	7.9 ± 2.6	2.9 ± 2.3	11.6 ± 2.4	2.2 ± 2.4	21.3 ± 2.4	24.4 ± 2.3
USN	1984	50.0 ± 2.5	4.7 ± 2.3	6.3 ± 2.2	10.7 ± 2.2	1.9 ± 2.2	16.5 ± 2.2	27.0 ± 2.2
	1990	46.5 ± 2.7	5.5 ± 2.3	5.2 ± 2.3	11.3 ± 2.3	2.0 ± 2.3	16.4 ± 2.3	26.8 ± 2.3
	1993	48.3 ± 3.2	4.4 ± 2.6	6.0 ± 2.4	11.2 ± 2.5	2.6 ± 2.5	16.4 ± 2.4	26.3 ± 2.4
IVS	1984	54.8 ± 3.0	9.4 ± 2.5	1.7 ± 2.2	6.3 ± 2.2	6.9 ± 2.2	17.3 ± 2.1	18.3 ± 2.2
	1990	54.0 ± 3.2	9.8 ± 2.7	1.9 ± 2.3	6.3 ± 2.3	6.8 ± 2.3	17.3 ± 2.3	18.2 ± 2.3
	1993	54.5 ± 3.6	9.3 ± 2.9	1.8 ± 2.4	6.1 ± 2.4	7.1 ± 2.4	17.3 ± 2.4	18.2 ± 2.4
C04	1984	52.4 ± 2.8	16.1 ± 2.4	8.1 ± 2.0	10.2 ± 2.1	6.4 ± 2.0	27.1 ± 2.0	24.5 ± 1.9
	1990	51.4 ± 3.1	16.4 ± 2.6	8.6 ± 2.2	11.3 ± 2.2	6.2 ± 2.1	27.2 ± 2.1	24.2 ± 2.1
	1993	48.8 ± 3.8	17.7 ± 3.0	8.9 ± 2.3	12.8 ± 2.3	6.8 ± 2.2	27.3 ± 2.2	24.4 ± 2.2
NEOS	1984	35.6 ± 2.1	21.1 ± 2.0	28.4 ± 1.9	16.4 ± 2.0	7.7 ± 1.9	14.9 ± 1.9	34.9 ± 1.9
	1990	31.9 ± 1.4	20.7 ± 1.4	16.5 ± 1.3	16.5 ± 1.3	4.0 ± 1.3	17.7 ± 1.3	28.0 ± 1.3
	1993	33.0 ± 1.5	20.7 ± 1.4	18.0 ± 1.3	16.4 ± 1.3	4.1 ± 1.3	16.2 ± 1.3	29.1 ± 1.3
Average		43.9 ± 2.9	14.2 ± 1.8	10.4 ± 2.1	12.6 ± 1.1	4.5 ± 0.7	19.2 ± 1.2	25.8 ± 1.2
					dY			
GSF	1984	57.0 ± 2.6	21.9 ± 2.3	7.3 ± 2.2	21.1 ± 2.2	5.1 ± 2.1	17.6 ± 2.1	9.3 ± 2.2
	1990	59.9 ± 2.8	21.3 ± 2.4	6.6 ± 2.3	21.2 ± 2.3	4.7 ± 2.2	17.5 ± 2.3	9.7 ± 2.3
	1993	62.2 ± 3.4	19.8 ± 2.7	7.9 ± 2.4	21.5 ± 2.4	5.0 ± 2.4	17.8 ± 2.4	10.4 ± 2.4
USN	1984	52.0 ± 2.6	11.5 ± 2.5	8.7 ± 2.3	20.4 ± 2.3	6.3 ± 2.3	16.8 ± 2.3	8.0 ± 2.3
	1990	50.3 ± 2.7	11.4 ± 2.5	8.1 ± 2.3	20.8 ± 2.4	5.5 ± 2.4	16.6 ± 2.3	8.4 ± 2.4
	1993	54.9 ± 3.3	8.5 ± 2.8	8.9 ± 2.5	20.9 ± 2.5	6.4 ± 2.5	16.6 ± 2.5	8.9 ± 2.5
IVS	1984	44.9 ± 2.7	22.1 ± 2.4	8.1 ± 2.1	24.6 ± 2.1	12.1 ± 2.1	17.5 ± 2.1	10.8 ± 2.1
	1990	44.7 ± 2.9	22.2 ± 2.6	8.2 ± 2.2	24.7 ± 2.3	12.1 ± 2.2	17.5 ± 2.2	10.9 ± 2.2
	1993	43.9 ± 3.0	22.9 ± 2.8	8.4 ± 2.3	24.7 ± 2.4	12.2 ± 2.3	17.6 ± 2.3	11.1 ± 2.3
C04	1984	55.6 ± 2.4	22.2 ± 2.1	6.2 ± 1.8	11.2 ± 1.8	7.4 ± 1.8	19.4 ± 1.8	16.0 ± 1.8
	1990	53.8 ± 2.4	25.0 ± 2.1	5.1 ± 1.9	10.9 ± 1.8	8.3 ± 1.9	19.4 ± 1.8	15.6 ± 1.8
	1993	59.2 ± 3.2	20.1 ± 2.5	5.4 ± 1.9	11.3 ± 1.9	7.6 ± 1.9	19.2 ± 1.8	15.2 ± 1.9
NEOS	1984	63.9 ± 2.6	14.2 ± 2.4	14.3 ± 1.9	25.9 ± 1.9	16.7 ± 1.8	12.6 ± 1.8	9.5 ± 1.8
	1990	60.1 ± 2.1	25.9 ± 2.1	3.7 ± 1.5	23.4 ± 1.5	16.3 ± 1.4	13.7 ± 1.4	12.1 ± 1.4
	1993	69.5 ± 2.1	17.9 ± 2.0	5.7 ± 1.4	25.5 ± 1.4	17.7 ± 1.3	14.1 ± 1.3	13.5 ± 1.3
Average		56.2 ± 2.1	19.6 ± 1.5	7.2 ± 0.9	20.6 ± 1.5	11.0 ± 1.4	16.5 ± 0.8	11.9 ± 0.8

Table 4. Corrections to the amplitudes of the nutation terms determined jointly with the corrections to the quadratic precession model, in μas

Series	T_1	Harmonic period, days						
		6798.38	3399.19	365.26	182.62	121.75	27.55	13.66
					dX			
GSF	1984	62.9 ± 3.1	15.9 ± 2.5	2.9 ± 2.1	11.9 ± 2.2	1.9 ± 2.2	21.7 ± 2.2	24.5 ± 2.1
	1990	55.0 ± 4.6	10.8 ± 2.8	2.1 ± 2.2	11.8 ± 2.3	1.9 ± 2.2	21.5 ± 2.2	24.3 ± 2.2
	1993	75.5 ± 10.2	13.3 ± 3.4	3.1 ± 2.3	12.2 ± 2.4	1.9 ± 2.4	21.4 ± 2.3	24.5 ± 2.3
USN	1984	48.4 ± 3.0	8.2 ± 2.4	6.0 ± 2.2	10.6 ± 2.2	1.9 ± 2.2	16.6 ± 2.2	27.1 ± 2.2
	1990	46.8 ± 4.7	6.8 ± 2.8	5.2 ± 2.3	11.3 ± 2.3	2.0 ± 2.3	16.5 ± 2.3	26.8 ± 2.3
	1993	66.7 ± 10.8	13.2 ± 3.6	6.2 ± 2.4	11.7 ± 2.5	2.3 ± 2.5	16.4 ± 2.4	26.4 ± 2.4
IVS	1984	66.4 ± 5.5	14.9 ± 2.8	2.1 ± 2.2	7.0 ± 2.2	6.5 ± 2.2	17.3 ± 2.1	18.5 ± 2.2
	1990	69.3 ± 7.4	15.9 ± 3.1	2.2 ± 2.3	7.1 ± 2.3	6.4 ± 2.3	17.3 ± 2.3	18.4 ± 2.3
	1993	102.2 ± 12.1	23.1 ± 3.7	2.4 ± 2.4	7.7 ± 2.4	6.2 ± 2.4	16.9 ± 2.4	18.6 ± 2.4
C04	1984	52.8 ± 3.7	16.2 ± 2.6	7.9 ± 2.0	10.3 ± 2.1	6.3 ± 2.0	27.1 ± 2.0	24.5 ± 1.9
	1990	57.0 ± 5.6	17.1 ± 2.8	8.3 ± 2.2	11.5 ± 2.2	6.0 ± 2.1	27.2 ± 2.1	24.1 ± 2.1
	1993	66.5 ± 13.6	16.5 ± 3.1	8.7 ± 2.3	12.9 ± 2.3	6.6 ± 2.2	27.2 ± 2.2	24.2 ± 2.2
NEOS	1984	40.8 ± 2.4	19.7 ± 2.1	28.4 ± 1.9	16.6 ± 1.9	7.7 ± 1.9	15.1 ± 1.9	34.9 ± 1.9
	1990	38.8 ± 2.2	23.0 ± 1.4	16.2 ± 1.3	16.4 ± 1.3	3.8 ± 1.3	17.8 ± 1.3	28.1 ± 1.3
	1993	101.7 ± 4.9	22.6 ± 1.8	17.4 ± 1.3	15.3 ± 1.3	3.8 ± 1.2	16.5 ± 1.2	29.1 ± 1.2
Average		51.7 ± 4.2	17.4 ± 1.5	10.3 ± 2.1	12.6 ± 1.0	4.4 ± 0.7	19.3 ± 1.2	25.9 ± 1.2
					dY			
GSF	1984	58.9 ± 3.6	21.0 ± 2.4	7.1 ± 2.2	21.0 ± 2.2	5.1 ± 2.1	17.6 ± 2.1	9.3 ± 2.2
	1990	60.0 ± 4.8	21.9 ± 2.7	6.7 ± 2.3	21.1 ± 2.3	4.7 ± 2.2	17.5 ± 2.3	9.7 ± 2.3
	1993	109.9 ± 10.8	26.7 ± 3.5	7.6 ± 2.4	20.0 ± 2.4	4.7 ± 2.4	18.1 ± 2.4	10.2 ± 2.4
USN	1984	51.6 ± 3.1	11.9 ± 2.5	9.0 ± 2.3	20.4 ± 2.3	6.3 ± 2.3	16.7 ± 2.3	8.0 ± 2.3
	1990	50.1 ± 4.9	11.4 ± 2.6	8.1 ± 2.3	20.8 ± 2.4	5.5 ± 2.4	16.6 ± 2.3	8.4 ± 2.4
	1993	114.3 ± 11.3	19.8 ± 3.7	8.6 ± 2.4	19.3 ± 2.5	5.7 ± 2.5	16.9 ± 2.5	8.7 ± 2.5
IVS	1984	59.1 ± 5.4	23.2 ± 2.7	8.4 ± 2.1	24.0 ± 2.1	12.2 ± 2.1	17.6 ± 2.1	11.0 ± 2.1
	1990	69.3 ± 7.6	24.1 ± 3.0	8.5 ± 2.2	23.7 ± 2.3	12.2 ± 2.2	17.6 ± 2.2	11.1 ± 2.2
	1993	103.8 ± 12.2	26.5 ± 3.2	8.8 ± 2.3	22.9 ± 2.4	12.2 ± 2.3	17.9 ± 2.3	11.4 ± 2.3
C04	1984	58.6 ± 3.0	23.3 ± 2.2	6.4 ± 1.8	11.0 ± 1.8	7.4 ± 1.8	19.3 ± 1.8	16.2 ± 1.8
	1990	46.5 ± 4.6	22.6 ± 2.4	5.0 ± 1.9	11.0 ± 1.8	8.4 ± 1.9	19.5 ± 1.8	15.2 ± 1.8
	1993	75.2 ± 11.0	19.6 ± 2.6	5.5 ± 1.9	11.2 ± 1.9	7.5 ± 1.9	19.2 ± 1.8	15.4 ± 1.9
NEOS	1984	63.5 ± 2.6	17.6 ± 2.7	14.5 ± 1.9	26.0 ± 1.9	16.7 ± 1.8	12.7 ± 1.8	9.6 ± 1.8
	1990	53.4 ± 3.1	23.7 ± 2.1	4.6 ± 1.5	23.4 ± 1.5	16.1 ± 1.4	14.1 ± 1.4	12.3 ± 1.4
	1993	89.6 ± 5.7	20.1 ± 2.1	4.7 ± 1.4	25.3 ± 1.4	17.9 ± 1.3	14.2 ± 1.3	13.4 ± 1.3
Average		59.9 ± 3.4	20.7 ± 1.3	7.2 ± 0.9	20.3 ± 1.5	10.9 ± 1.4	16.6 ± 0.8	11.9 ± 0.8

are appreciably smaller than those for the parameters of the quadratic model; this is not surprising, given the comparatively short observational interval. Moreover, the results obtained using the linear precession model and the different observational intervals essentially coincide, which is not true of the quadratic precession model.

Note the large shift parameters obtained, especially in the coordinate Y. Although the mean correction for the coordinate X is modest, this is due to the results for the NEOS series, which differ sharply from the other results. The results for this series for all other parameters correspond to those for the other data.

When applying the quadratic model, the corrections to the precession rate p_1 and the quadratic term p_2 were uncertain. Although the formal mean correction to p_1 is statistically significant, the large scatter in the values obtained for the different data series leads us to believe there are no clear grounds for recommending that the precession rate in the IAU 2000/2006 precession model be corrected.

Tables 3 and 4 present the calculated corrections to the amplitudes of the seven main nutation terms, determined jointly with the corrections for the linear and quadratic precession models. We obtained appreciable and statistically significant corrections for the amplitudes of both the long-period and short-period nutation terms. There is overall a fairly good agreement for the results obtained using the different data series, with the exception of the nutation term with a period of 18.6 years. It can be also noted that a better accuracy in the amplitudes of the long-period nutation terms is obtained using the linear model for the corrections to the precession parameters, as well as a smaller scatter in the values obtained for the different observational intervals. It is obvious that the duration of the available interval of VLBI observations is still insufficient to distinguish this nutational oscillation from the polynomial terms describing the precession. The corrections to the amplitudes of the nutation terms with periods of a year or less are virtually independent of the precession-correction model used and, in most cases, also independent of the observational interval.

The random errors in the corrections to the IAU 2000/2006 also depend appreciably on both the CPO series and the observational interval used, with this dependence being different for different parameters. The errors in the corrections to the precession parameters and the long-period (6798.38 and 3399.19 day) nutation terms grow as the duration of the observational interval is decreased; i.e., as the start of this interval is shifted from 1984.0 to 1993.0. We must also take into account that the growth in the errors in the precession parameters is determined not only by the decrease in the observational interval and the number of data, but also by the increase in the shift of the standard epoch of the theory 2000.0 relative to the middle of this interval (it is known that, when determining the parameters of a trend using a least-squares fit, the errors in the resulting estimates are smallest when the the initial epoch coincides with the middle of the interval). At the same time, the errors in the amplitudes of the nutation terms with periods of a year or less depend only weakly on the observational interval used, which can be explained by the compensation of the decrease in the number of data by the increase in their accuracy when the early observations are excluded. The errors in the amplitudes of all the nutation terms for the NEOS series apart from the 18-year term decrease appreciably when the start of the interval is shifted from 1984.0 to 1993.0. Finally, when choosing the optimal interval for the VLBI observations, we must take into account the large systematic errors in the data obtained in the middle of the 1980s, which are clearly visible in Fig. 1, especially for dY.

4. CONCLUSION

The accuracy of models for precession and nutation are determined by the accuracy of the astronomical and geophysical models applied in their construction, as well as the accuracy of the CPO series used to refine the model parameters. A comparison of the corrections to the main parameters of the IAU 2000/2006 precession and nutation model derived using five individual and combined CPO series has shown that these depend appreciably on the CPO data series and observational interval used. The results of our analysis do not provide convincing arguments as to whether the use of individual or combined data series is preferable. On the other hand, such comparisons provide important material for the analysis of systematic errors in the CPO series, and thus can aid in enhancing their accuracy.

Overall, our results indicate that the official values of most of the key parameters of the IAU 2000/2006 model, first and foremost the shift in the pole and the amplitudes of the main nutation terms, require significant correction. However, the values for these corrections derived for different CPO series differ appreciably. Therefore, it is expedient to determine a set of final corrections to the precession and nutation models using more than a single CPO series, and based on a thorough comparison, and possibly averaging, of results obtained using a variety of data sets and observational periods. The use of a small number of selected series without carrying a special comparative analysis, as has virtually always been done thus far, could lead to substantial systematic errors in the Earth-rotation parameters.

Our calculations demonstrate that the results for the precession parameters and the parameter of the long-period nutation terms depend appreciably on the adopted model for the corrections to the precession parameters. Our results show that applying the quadratic model significantly lowers the accuracy with which all these parameters are determined. In addition, the scatter in the results obtained for different series and observational intervals grows appreciably. It is thus obvious that the available interval of VLBI observations is sufficient to reliably improve only the shift in the celestial pole and the precession rate, i.e., only the parameters of the linear precession model.

ACKNOWLEDGMENTS

The author thanks the referee for useful comments, which have appreciably improved this paper.

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Translated by D. Gabuzda